Dual Dynamics theory

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A theory based on dual dynamics (propagation and confinement) is proposed in a mathematical framework including a redefinition of quantum states, creation and annihilation operators, fermions and bosons distinction but also of colored charges and spins. Particles interactions are found to be either direct (fermion-fermion) or indirect (mediated by bosons), as a consequence of a revisited wave-particle duality. Fundamental interactions as well as elementary particles naturally emerge from the dual driving equations applied to vector potential states. They are qualitatively compared to the content of the Standard Model, evidencing some interesting features such as confinement, hierarchy, and parity violation. Introducing nonlinear coupling terms further allows the appearance of a photon wave function and a "composite graviton field" and is foreseen to produce generations of particles through a self trapping mechanism. In the last part, cosmology is analyzed in the framework of the dual dynamics theory. The non-linearities generate Bose-Einstein condensates leading to black-holes through attracting potentials. Quasars and blazars also emerge with the introduction of "jet-particles" originating from the Legendre function of the second kind. Non-baryonic matter finally shows up in the present theory. It can form "dark" Bose-Einstein condensate creating halos around black-holes. A new definition of the equivalence principle between inertial and gravitational masses is proposed allowing anti-particles to have negative gravitational masses without violating the usual test experiments. This renews the concept of anti-gravity, which plays the role of dark-energy in the present theory. Finally the universe time-line is envisioned in the context of the coupled and nonlinear dual equations (propagation and confinement), requiring to revisit the Big-Bang and inflation mechanisms, the latter being attributed to a superluminal expansion, which is allowed by the nonlinear terms.

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1 Introduction

The Standard Model [1] and the general relativity [2] allow explaining many physical properties from the quark scale to the universe scale with very high accuracy. Yet, many questions remain unsolved in the framework of the two theories considered separately [3]. The most intriguing one is that only ≈5% of the overall mass is explained by standard theories, the remaining ≈95% being included in dark matter and dark energy [4], underlying our current ignorance on their very nature. Gathering all fundamental forces and particles in a more general and unified theory appears as a possible and probably necessary route to overcome these difficulties. Many attempts have been made so far, the most famous being string theories, super-symmetries and quantum gravity. All these theories offer many new and interesting features likely to solve part of the encountered issues, but to date a unified theory is still missing. In this work, new postulates are proposed, implying redefinitions of many well established concepts such as quantum states, creation and annihilation operators, fermions and bosons, colored charges, spins, etc. But perhaps more importantly it requires new driving equations generating "dual" dynamics (propagation and confinement). Hence, after having introduced the basics of the DuDy (for dual dynamics) theory in section 2, the manuscript is divided into three parts corresponding to the investigation of particle physics (section 3) and cosmology (section 5) with an intermediate step for discussing the implication of non-linearities in the driving equations (section 4).

Before starting, I would like to warn the reader on the fact that, belonging to any of the communities dealing with what follows, but mainly working on nonlinear optical properties of nano-particles, this work is completely outside the framework of my daily research. A side effect is that the language and mathematics used in this work may differ from the academic ones. Thought, the present DuDy theory offering interpretations for some unsolved problems in physics [3], I've decided to reveal the present work to the scientific community, expecting that, if not fully correct, the DuDy theory may help tracing new routes toward a unified description of physics.

2 Basics of DuDy theory

2.1 General covariance

General covariance traduces the invariance of the form of physical laws under arbitrary differentiable coordinate transformations. One of the most famous attempt is the pioneering work of Einstein on special [5] and general relativity [2], where space and time appear as a unified space-time entity governed by Lorentz transformations [6]. Despite the conceptual aesthetics and success of Einstein theory in describing many experimental results, a completely different approach will be followed in the present theory. A flat space-time will be used in association with Galilean transformations. It is worth noting that making this unusual choice will highly constrain the present theory as, at the end of the derivation, one has to recover for example the famous relation $E^2 = p^2c^2 + m^2c^4$ of Einstein special relativity [5] or the fact that photons can be deviated by massive stars [2], for example.

For discussing how general covariance will be handled, let us consider the simple case of a classical string described by plane waves

$$\phi\left(\vec{r}, t\right) = \phi_0 \exp\left[i\left(\vec{k}.\vec{r} - \omega t\right)\right] \tag{1}$$

where $\vec{k}$ is the wave-vector and ω the frequency. The final goal being to derive physics laws that are independent of a given (arbitrary) choice of coordinate transformation (Galilean, Lorentz,...), ***it is postulated that the wave-functions are the same in all frame of reference***. As a consequence, a first well-known general invariant quantity associated with the wave-function phase emerges:

$$\vec{k}.\vec{r} - \omega t. \tag{2}$$

Depending on the used coordinate transformation, the wave vector $\vec{k}$, the pulsation ω or the phase velocity v might be "accidentally" an invariant, but $\vec{k}.\vec{r} - \omega t$ ***has to be independent of the frame of reference and more importantly independent of the used coordinate transformation. Therefore, all observable quantities must involve only terms of this kind*** in order to ensure the general covariance of the physical laws. This is indeed not the choice usually made in quantum mechanics. In order to illustrate this discussion, let us consider the standard second quantification procedure applied to the classical string. Introducing the creation $\tilde{a}_n^\dagger$ and annihilation $\tilde{a}_n$ operators (the ˜ symbol is used since these operators will be modified later on), the string wave function takes the usual form [6]:

$$\tilde{\phi} = \sum_{n=-\infty}^{n=+\infty} \sqrt{\frac{\hbar}{2\omega_n \mu l}} \left\{\tilde{a}_n e^{i(k_n z - \omega_n t)} + \tilde{a}_n^\dagger e^{i(-k_n z + \omega_n t)}\right\} \tag{3}$$

where l is the string length and μl the string mass. The subscript n labels different eigenstates, having a pulsation $\omega_n > 0$ and a wave-vector k_n ($k_n < 0$ for $n < 0$). Injecting this solution into the Hamiltonian associated with a classical string, one recovers the well known expression [6]

$$\tilde{H} = \sum_{n=-\infty}^{n=+\infty} \hbar\omega_n (\tilde{a}_n^\dagger \tilde{a}_n + 1/2). \tag{4}$$

Clearly, ***neither the wave-function $\tilde{\phi}$ ($\propto 1/\sqrt{\omega_n}$) nor the Hamiltonian $\tilde{H}$ ($\propto \omega_n$) are independent of the frame of reference and of the coordinate transformation***, since ω_n is not an invariant quantity in Galilean transformations and in Lorentz transformations [6]. Both of them have to be modified since a general covariance is requested for the present theory. For this purpose, the wave-function ϕ is redefined as follows

$$\phi = \sum_{n=-\infty}^{n=+\infty} \phi_n = \sum_{n=-\infty}^{n=+\infty} \frac{1}{\sqrt{2l}} \left\{ \tilde{a}_n e^{i(k_n z - \omega_n t)} + \tilde{a}_n^\dagger e^{i(-k_n z + \omega_n t)} \right\} \tag{5}$$

and the following forms are postulated for Hamiltonians and Lagrangians

$$\begin{aligned} H_n &= \int_0^l \left\{ \frac{1}{2} \left(\frac{\partial \phi_n}{\omega_n \partial t} \right)^2 + \frac{1}{2} \left(\frac{v}{\omega_n} \frac{\partial \phi_n}{\partial z} \right)^2 \right\} dz \\ L_n &= \int_0^l \left\{ \frac{1}{2} \left(\frac{\partial \phi_n}{\omega_n \partial t} \right)^2 - \frac{1}{2} \left(\frac{v}{\omega_n} \frac{\partial \phi_n}{\partial z} \right)^2 \right\} dz \end{aligned} \tag{6}$$

where v corresponds to the phase velocity. Basically, the wave-function has been multiplied by $(\mu\omega_n)^{1/2}$ whereas the Hamiltonian and the Lagrangian have been divided by $\mu\omega_n^3$ with ω_n depending on the considered eigenmode. Doing this, the wave-function has now a generally covariant form since the phase term $\vec{k}.\vec{r} - \omega t$ is an invariant quantity. ***The condition for both Hamiltonian and Lagrangian to be also generally covariant gives the second invariant quantity***:

$$\frac{k_n^2 v^2}{\omega_n^2}. \tag{7}$$

This invariant is directly connected to the dispersion relation, namely $\omega_n^2 = k_n^2 v^2$, one can obtain by minimizing the Lagrangian or writing down the wave-equation. Using equation (6), the quantized expression of the Hamiltonian now reads

$$H = \sum_{n=-\infty}^{n=+\infty} (\tilde{a}_n^\dagger \tilde{a}_n + 1/2). \tag{8}$$

This is a key ingredient in the DuDy theory: the energy needed to create a particle does not depend on an arbitrarily chosen frame of reference nor on a coordinate transformation. Here, a physical particle intrinsically exists with a given -universal- energy and wave-function.

Interestingly, the Hamiltonian H in Eq. 8 is directly linked to the quantized number of particles $\tilde{N}_n = \tilde{a}_n^\dagger \tilde{a}_n$ in this description. ***The general covariance of the Hamiltonian*** H ***is therefore equivalent to the general covariance of particle numbers***. In this context, the energy conservation has to be compared with the conservation of leptons or quarks numbers, for example. Nevertheless, the usual energy conservation law

$$\sum_i \hbar\omega_i = \sum_f \hbar\omega_f \tag{9}$$

is still valid in the present approach. It arises from the fact that the transition probability amplitude between initial and final states is proportional to

$$\int_{-\infty}^{+\infty} \exp\left[i\left(\sum_f \omega_f - \sum_i \omega_i\right)t\right] dt, \tag{10}$$

and vanishes unless $\sum_i \omega_i = \sum_f \omega_f$. This conservation law is therefore a consequence of the time invariance (as well as the impulsion conservation is a consequence of the space invariance) and its very nature differs from that of the particle number conservation law.

2.2 Creation and annihilation of identical particles

Within the second quantification, both modified (Eq. 8) and original (Eq. 4) Hamiltonians involve expressions such as $\tilde{a}_n^\dagger \tilde{a}_n + 1/2$ usually written as $\tilde{N}_n + 1/2$. The latter form is especially interesting as it allows a direct interpretation in terms of number of bosons N_n as discussed before. However, the additional 1/2 leads to an infinite expectation value for the vacuum energy. It has motivated the use of "normal ordered product" $:H:$ which is obtained by subtracting the vacuum expectation value $\langle 0|H|0\rangle$ from the Hamiltonian H [6]. As we will see, this plays an important role in the identification of identical (indiscernible) particles as being fermions or bosons.

The indiscernibility of particles can be expressed by the fact that exchanging indiscernible particles does not change the expectation value for any operator. Introducing the Fock state $|1_n 1_m\rangle$ of two indiscernible particles, this reads [6]

$$|\langle f|O|1_n 1_m\rangle|^2 = |\langle f|O|1_m 1_n\rangle|^2 \tag{11}$$

where O is an operator and $|f\rangle$ a given final state. Hence, one can show that $|1_n 1_m\rangle$ and $|1_m 1_n\rangle$ differ only by a phase term $e^{i\delta}$. Applying twice the permutation gives the original state, so that $\delta = 0[\pi]$ or equivalently

$$|1_n 1_m\rangle = \pm|1_m 1_n\rangle. \tag{12}$$

This "$\pm$" sign indicates that there are two types of identical particles, giving either a plus or a minus sign while permuting two of them. Interestingly, this simple rule leads to strong constraints on the creation and annihilation operators which are divided into two classes depending on the balance they obey [6]:

- $\tilde{a}_n^\dagger \tilde{a}_n^\dagger = 0$ for particles which cannot be in the same eigenstate, *i.e.* fermions associated with Fermi-Dirac statistic,
- $\tilde{a}_n^\dagger \tilde{a}_n^\dagger \neq 0$ for particles which can exist in the same state, *i.e.* bosons driven by Bose-Einstein statistic.

This result is of great importance since the distinction between fermions and bosons naturally emerges from these considerations (as well as the Pauli principle for fermions).

In addition to their specific statistics, fermions and bosons also differ by the Hamiltonians or Lagrangians describing their dynamics, as for example, the Klein-Gordon and Dirac equations dedicated to spin-less and spin 1/2 particles respectively. Though, these Hamiltonians can be quantized using the same form [6]

$$: H := H - \langle 0|H|0\rangle = \sum_{n=-\infty}^{n=+\infty} E_n(\tilde{a}_n^\dagger \tilde{a}_n + \tilde{c}_n^\dagger \tilde{c}_n) \tag{13}$$

where $\tilde{c}_n^\dagger$ and $\tilde{c}_n$ are interpreted as creation and annihilation of anti-particles. Enforcing $: H :$ to be positive leads to the right hand side of Eq. 13 provided that [6]

- anti-commutators $\left\{\tilde{a}_n \tilde{a}_m^\dagger\right\} = \delta_{n,m}$ are chosen for Dirac equation, *i.e.* Dirac equation describes fermions,
- commutators $\left[\tilde{a}_n \tilde{a}_m^\dagger\right] = \delta_{n,m}$ are used for Klein-Gordon equation, *i.e.* Klein-Gordon equation stems for bosons.

This fundamental result is often considered as a major success of field theories to explain particle physics as it provides the missing link between spin, statistics and dynamics. It is nevertheless puzzling that the infinite expectation value $\langle 0|H|0\rangle$ has to be removed from the Hamiltonian in order to reach this conclusion.

Instead of using the normal ordered product, a different strategy is followed in the present theory, leading to a new definition of the creation and annihilation operators and consequently of the quantum states. Here again, since one has to obtain at the end of the derivation the same kind of expression as in Eq. 13, the choice allowed for these new operators and wave-functions is highly constrained. Using notations similar to the previous ones for the new creation $a_n^\dagger$ and annihilation a_n operators, the following relations are postulated:

$$\begin{aligned} \langle 1_n|1_m\rangle &= \delta_{n,m} \\ a_n^\dagger|N_n\rangle &= |N_n+1\rangle \\ a_n|N_n\rangle &= |N_n-1\rangle \\ |N_n\rangle &= \sqrt{N_n}.|1_n\rangle. \end{aligned} \tag{14}$$

These relations are at variance with the usual ones, leading to many consequences:

- first, ***the quantum states are no more normalized but scale as the square root of the number of particles***. This surprising result in view of the standard second quantification is, indeed, conform to what can be expected for a quantized energy proportional to the squared wave-function: if for example the classical electromagnetic energy is doubled, the electric field amplitude is multiplied by $\sqrt{2}$.

- second, ***two quantum states having the same*** n ***(same wave vector) but different occupation numbers*** N_n ***are no more orthogonal***, again in agreement with classical electromagnetism.

- third, ***creation and annihilation operators commute irrespective to the nature of the particles (fermions or bosons)***:

$$a_n^\dagger a_n|N_n\rangle = a_n a_n^\dagger|N_n\rangle = |N_n\rangle. \tag{15}$$

  This means that, in contrast with quantum field theories [6], the Pauli exclusion principle as well as the distinction between fermions and bosons has to find its origin elsewhere, *i.e.* not in the definition of creation and annihilation operators, as it will be demonstrated in the section 3.2.

- fourth, ***creation and annihilation operators are non-linear operators*** as can be easily deduced from Eqs. 14 since $a_n^\dagger\sqrt{N_n}|1_n\rangle \neq \sqrt{N_n}a_n^\dagger|1_n\rangle$. However $a_n^\dagger a_n$ and $a_n a_n^\dagger$ are still linear operators equal to Identity as seen from Eq. 15.

To conclude this part on creation and annihilation operators, the expression of the Hamiltonian of a classical string is derived as discussed in section 2.1. Using the wave-function given in Eq. 5:

$$\phi = \sum_{n=-\infty}^{n=+\infty} \frac{1}{\sqrt{2l}} \left\{a_n e^{i(k_n z-\omega_n t)} + a_n^\dagger e^{i(-k_n z+\omega_n t)}\right\}, \tag{16}$$

the new quantized Hamiltonian is

$$H = \sum_{n=-\infty}^{n=+\infty} a_n^\dagger a_n. \tag{17}$$

As a consequence and as expected, the vacuum expectation value $\langle 0|H|0\rangle$ is therefore zero in the present theory, since $\langle 0|0\rangle = 0$, without invoking the normal ordered products. However, as underlined previously, the "price to pay" is to find an alternative way to differentiate fermions and bosons. Before entering in this discussion, which will constitute the introduction in the particle physics as viewed through the present theory, one needs to establish the last and most important ingredient of the DuDy theory.

2.3 Dual Dynamics

The wave-functions discussed in the previous section are solutions of the wave-equation:

$$\left(\frac{\partial^2}{c^2\partial t^2} - \Delta\right)\phi = 0. \tag{18}$$

Using the standard transformation $p \leftrightarrow -i\hbar\vec{\nabla}$ and $E \leftrightarrow i\hbar\frac{\partial}{\partial t}$ one obtains

$$-\frac{E^2}{c^2} + p^2 = 0 \tag{19}$$

relating the particle energy E to its linear momentum p. This expression is valid for a mass-less particle but deviates from the well-known expression of special relativity [5]:

$$-\frac{E^2}{c^2} + p^2 + m^2c^2 = 0, \tag{20}$$

This indicates that some physics is missing when the particle mass m is non-zero. This issue has been solved in the past by modifying the wave-equation, leading to the Klein-Gordon equation [6]:

$$\left(\frac{\partial^2}{c^2\partial t^2} - \Delta + \frac{m^2c^2}{\hbar^2}\right)\phi = 0. \tag{21}$$

This form is interpreted as the relativistic field equation for a scalar (i.e. spin-0) particle which can be solved using plane waves. Another approach was proposed by Dirac [6], based on the fact that

$$\frac{\partial^2}{c^2\partial t^2} - \Delta = \left(A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y} + C\frac{\partial}{\partial z} + iD\frac{\partial}{c\partial t}\right)\left(A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y} + C\frac{\partial}{\partial z} + iD\frac{\partial}{c\partial t}\right) \tag{22}$$

where A, B, C and D are 4x4 matrices squaring to the 4x4 identity. It provides a description of elementary half-spin particles, such as electrons, consistent with the principles of quantum mechanics and the theory of special relativity.

Here, we will proceed in a different way. ***Is is postulated that the particle dynamics is described by two coupled equations and wave-functions***. More precisely, the Klein-Gordon equation (Eq. 21) written for harmonic solutions

$$\left(-k^2 - \Delta + \frac{m^2c^2}{\hbar^2}\right)\phi = 0 \tag{23}$$

is split into the following equations:

$$\begin{aligned} &\text{Propagation} \quad \left(-k_p^2 - \Delta\right)\phi_p &= 0 \quad \text{where} \quad k_p^2 = \frac{2\omega_p^2}{c^2} \qquad (24)\\ &\text{Confinement} \quad \left(-k_c^2 + \Delta\right)\phi_c &= 0 \quad \text{where} \quad k_c^2 = \frac{2m^2c^2}{\hbar^2}. \qquad (25)\end{aligned}$$

This has many consequences on the interpretation of physical quantities:

- first, ***a given particle is no more described by a single wave-function as done in quantum mechanics but by a couple of wave-functions*** ϕ_p ***and*** ϕ_c. In that sense, these wave-functions cannot be anymore interpreted as probability amplitudes. Note that this renouncement in a direct probabilistic interpretation has already been undertaken with the previous description of creation and annihilation operators (cf. 2.2), since the quantum states are no more normalized to unity.

- second, Eq. 24 corresponds to the usual propagation equation associated with mass-less particles like photons. Its solutions are often expressed in free space as harmonic eigenmodes $e^{i(\vec{k}.\vec{r}-\omega t)}$. Exchanging $k \leftrightarrow ik$ transforms Eq. 24 into Eq. 25 and gives to the solutions of Eq. 25 an exponentially increasing or decreasing character. These solutions are generally disregarded in free space owing to their diverging behaviors. In contrast, they usually appear in evanescent waves, corresponding to a strong spatial localization. For this reason, ***Eq. 25 will be referred as the "confinement" equation, giving to the present theory its grounding as being governed by Dual Dynamics (DuDy)***.

- third, ***since DuDy theory involves dual wave-equations, it also requires dual Hamiltonians*** as it is the case in super-symmetric (SuSy) theories [1]. However, in the present case, these two equations are not associated with fermions and bosons having the same mass and internal quantum numbers. It rather refers to two classes of wave-functions providing each different physical properties, namely propagation (Eq. 24) and confinement (Eq. 25), to the same particle state. Hence, their oscillation frequency will be assumed to be the same: $\omega_p = \omega_c = \omega$ and $k_p = k_c = k = \omega/c$. This has a profound consequence: ***the mass appears as a signature of an internal clock driven by the confinement equation***. If it is common to use the $E \rightarrow i\hbar\frac{\partial}{\partial t}$ transformation, exchanging $mc^2 \leftrightarrow i\hbar\frac{\partial}{\partial t}$ is a key ingredient in the present theory.

Allowing the use of generally forbidden mathematical solutions will appear as a "leitmotiv" in the approach presented in this work. The aim of the following discussion is therefore to make a first step in this direction by assuming that plane waves are not the best choice for describing massive and point-like particles. Instead, the presence of the particle at a given position is considered as breaking the space translation invariance. This suggests to use the spherical coordinates (r, θ, φ) in place of the Cartesian coordinates, giving a singular role to the referential origin where the particle will be located. Separating variables, the solutions of the propagation equation are well known as the spherical Bessel functions [7]

$$\begin{aligned} j_l(x) &= (-1)^l x^l \left(\frac{d}{xdx}\right)^l \left(\frac{\sin x}{x}\right) \\ n_l(x) &= -(-1)^l x^l \left(\frac{d}{xdx}\right)^l \left(\frac{\cos x}{x}\right) \end{aligned} \tag{26}$$

where $x = kr$. For the confinement equation, one might directly apply the transformation $k \leftrightarrow ik$, but it is custom to use spherical modified Bessel functions as these functions are real valued [7]

$$\begin{aligned} i_l(x) &= i^{-l} j_l(ix) \\ k_l(x) &= i^{(l+1)} n_l(ix). \end{aligned} \tag{27}$$

Note that a different choice is often made for $k_l(x)$ defined as $k_l(x) = -(i)^l h_l^{(1)}(ix)$, where $h_l^{(1)}(x) = j_l(x) + in_l(x)$ is a spherical Hankel function. The latter expression is less satisfactory as it has no definite parity and would obscure somehow the following discussion about the physical implication of spherical modified Bessel functions.

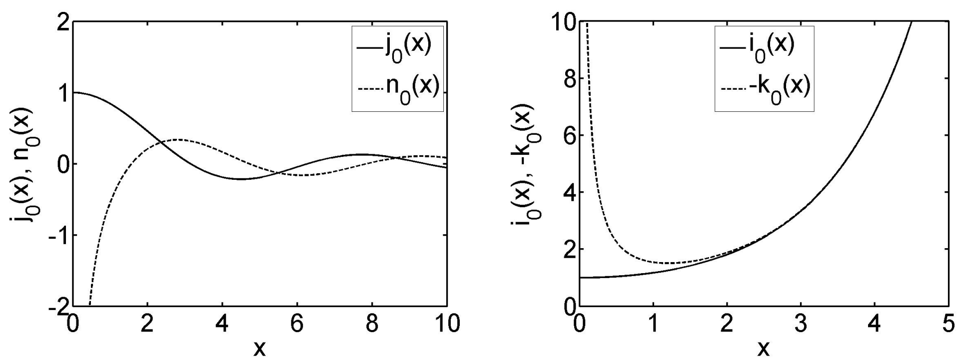

Figure 1: Spherical Bessel functions $j_l(x)$, $n_l(x)$ and modified spherical Bessel functions $i_l(x)$, $k_l(x)$ for $l = 0$.

As shown in Fig. 1 for $l = 0$, the $n_l(x)$, $i_l(x)$ and $k_l(x)$ functions are diverging functions either for $x \to 0$ or for $x \to +\infty$. This explains why they are usually rejected when these two asymptotic regions are involved, reducing the choice to the $j_l(x)$ functions which have a regular behavior. ***In the present theory,*** $\mathbf{n_l(x)}$***,*** $\mathbf{i_l(x)}$ ***and*** $\mathbf{k_l(x)}$ ***will indeed play a central role***. Let us therefore study their singular properties in more details. The asymptotic expansions of $i_l(x)$ and $k_l(x)$ for $x \to +\infty$ are given by [7]

$$i_l(x) \approx \frac{e^x}{2x} \quad \text{and} \quad k_l(x) \approx -\frac{e^x}{2x}. \tag{28}$$

Hence, despite the diverging character of both $i_l(x)$ and $k_l(x)$ for $x \to +\infty$, $i_l(x) + k_l(x)$ is indeed vanishing, i.e. there is an exact cancellation of the diverging term. Similarly, the asymptotic expansion of $n_l(x)$ and $k_l(x)$ for $x \to 0$ reads [7]

$$n_l(x) \approx -\frac{(2l)!}{2^l l!}\frac{1}{x^{l+1}} \quad \text{and} \quad k_l(x) \approx -\frac{(2l)!}{2^l l!}\frac{1}{x^{l+1}}. \tag{29}$$

Therefore the $1/x^{l+1}$ terms cancel out if one consider the right admixture of $n_l(x)$ and $k_l(x)$ functions. ***This exact cancellation of diverging terms reminds the philosophy of super-symmetric theories*** [1], although it will be used here in a different context.

3 DuDy and the particle physics

3.1 Driving equations

In the previous section was introduced the idea that a particle will no more be described by a single wave-function but by two separated contributions arising from the propagation and confinement equations (Eqs. 24-25). Here, ***it is further postulated that a particle, being associated with fermions or bosons, is described by a vector field*** $\vec{A}$, in analogy with the vector potential of classical electromagnetism, but at variance with the scalar wave-function of quantum mechanics. The consequences are numerous as we will see throughout the present manuscript. The first one is that the scalar wave-equations associated with propagation (Eq. 24) and confinement (Eq. 25) are replaced by vector wave-equations:

$$\left(\frac{\partial^2}{\omega^2\partial t^2} - \frac{1}{k^2}\Delta\right)\vec{A}_p = \vec{0} \tag{30}$$

$$\left(\frac{\partial^2}{\omega^2\partial t^2} + \frac{1}{k^2}\Delta\right)\vec{A}_c = \vec{0}. \tag{31}$$

Written is this way, the propagation equation (Eq. 30) is the same as the wave-equation deduced from the Maxwell's equations in vacuum, where the source terms have been removed. Hence, I would like to discuss in detail the connections between these two approaches. For a given a potential vector $\vec{A}_p$, three differential operators $\frac{\partial}{\omega\partial t}$, $\frac{1}{k}\vec{\nabla}\times$ and $\frac{1}{k}\vec{\nabla}.$ can be applied, defining three physical quantities:

$$\begin{aligned} \vec{E}_p &= -\frac{\partial\vec{A}_p}{\omega\partial t} \\ \vec{B}_p &= \frac{1}{k}\vec{\nabla}\times\vec{A}_p \\ D_p &= \frac{1}{k}\vec{\nabla}.\vec{A}_p. \end{aligned} \tag{32}$$

By analogy with Maxwell's equations, the two first terms will be called the electric field $\vec{E}_p$ and the magnetic field $\vec{B}_p$. ***It has to be noted that there is no scalar potential*** V_p***, which has been fixed to 0V and will therefore never appear in the present theory***. The longitudinal character of the electric field is entirely included in the vector potential $\vec{A}_p$. Applying the above differential operators on Eqs. 32 leads to three new quantities:

$$\begin{aligned} \vec{j}_{\partial t} &= \frac{\partial^2\vec{A}_p}{\omega^2\partial t^2} \\ \vec{j}_{\vec{\nabla}\times} &= \frac{1}{k^2}\vec{\nabla}\times\vec{\nabla}\times\vec{A}_p \\ \vec{j}_{\vec{\nabla}.} &= -\frac{1}{k^2}\vec{\nabla}.\left(\vec{\nabla}.\vec{A}_p\right). \end{aligned} \tag{33}$$

The first term of Eqs. 33 is known as the displacement current in Maxwell's equations. For this reason, the three vectors in Eqs. 33 will be called "currents" in the present theory. Interestingly, the real-valued harmonic solutions of the wave-equation (Eq. 30) are also eigenmodes of the "squared" operators $\frac{\partial^2}{\partial t^2}$, $\vec{\nabla}\times\vec{\nabla}\times$ and $\vec{\nabla}.\left(\vec{\nabla}.\ \right)$. These currents are therefore linked with temporal and spatial invariance properties. In this respect, the latter current, involving $\vec{\nabla}.$ terms, is singular since it leads first to a scalar function, corresponding to the divergence of the vector field, and then to a vector field given by the gradient of the scalar function. Hence, $\vec{j}_{\vec{\nabla}.}$ applies to the longitudinal modes only, whereas $\vec{j}_{\vec{\nabla}\times}$ is dedicated to transverse modes (the current $\vec{j}_{\partial t}$ applies to all solutions). Adding the three currents given in the Eqs. 33 leads to the following equation

$$\frac{\partial^2\vec{A}_p}{\omega^2\partial t^2}+\frac{1}{k^2}\vec{\nabla}\times\vec{\nabla}\times\vec{A}_p-\frac{1}{k^2}\vec{\nabla}.\left(\vec{\nabla}.\vec{A}_p\right)=\vec{j}_{\partial t}+\vec{j}_{\vec{\nabla}\times}+\vec{j}_{\vec{\nabla}.}=\vec{j}_p. \tag{34}$$

If one disregards the term $-\frac{1}{k^2}\vec{\nabla}.\left(\vec{\nabla}.\vec{A}_p\right)$, this balance is nothing else but the Maxwell-Ampère equation. Including this term, one recovers the propagation equation (Eq. 30) for both longitudinal and transverse modes in the presence of the source term $\vec{j}_p$

$$\left(\frac{\partial^2}{\omega^2\partial t^2}-\frac{1}{k^2}\Delta\right)\vec{A}_p=\vec{j}_p. \tag{35}$$

Is it also possible to recover the other Maxwell's equations? For this proposal, the second order derivatives associated with the operators $\frac{\partial}{\omega\partial t}$, $\frac{1}{k}\vec{\nabla}\times$ and $\frac{1}{k}\vec{\nabla}.$ have to be mixed in contrast with what was done in Eq. 33:

$$\begin{aligned}
&\frac{1}{k}\vec{\nabla}\times\vec{E}_p=-\frac{1}{k}\vec{\nabla}\times\left(\frac{\partial\vec{A}_p}{\omega\partial t}\right)=-\frac{\partial\vec{B}_p}{\omega\partial t}\\
&\frac{1}{k}\vec{\nabla}.\vec{B}_p=\frac{1}{k}\vec{\nabla}.\left(\frac{1}{k}\vec{\nabla}\times\vec{A}_p\right)=0\\
&\frac{1}{k}\vec{\nabla}.\vec{E}_p=\frac{1}{k}\vec{\nabla}.\left(-\frac{\partial}{\omega\partial t}\vec{A}_p\right)=\rho_{\partial t}
\end{aligned} \tag{36}$$

In the last equation, the notation $\rho_{\partial t}$ is somehow obscure at this step since, by analogy with the Maxwell's equations, the charge density ρ_p is expected. Indeed, as for the currents, the charge density will have different contributions. This can be seen by taking the divergence of Eq. 34:

$$\frac{1}{k}\vec{\nabla}.\left(\frac{\partial^2\vec{A}_p}{\omega^2\partial t^2}+\frac{1}{k^2}\vec{\nabla}\times\vec{\nabla}\times\vec{A}_p-\frac{1}{k^2}\vec{\nabla}.\left(\vec{\nabla}.\vec{A}_p\right)\right)=\frac{1}{k}\vec{\nabla}.\vec{j}_p. \tag{37}$$

Defining the following charge densities $\rho_{\partial t}$, $\rho_{\vec{\nabla}.}$ and $\rho_{\vec{\nabla}\times}$ by:

$$
\begin{aligned}
\frac{\partial}{\omega\partial t}\rho_{\partial t} &= -\frac{1}{k}\vec{\nabla}.\vec{j}_{\partial t} \\
\frac{\partial}{\omega\partial t}\rho_{\vec{\nabla}.} &= -\frac{1}{k}\vec{\nabla}.\vec{j}_{\vec{\nabla}.} \\
\frac{\partial}{\omega\partial t}\rho_{\vec{\nabla}\times} &= -\frac{1}{k}\vec{\nabla}.\vec{j}_{\vec{\nabla}\times} = 0
\end{aligned}
\tag{38}
$$

the previous equation can be recast into the charge conservation law

$$
\frac{\partial}{\omega\partial t}\rho_p + \frac{1}{k}\vec{\nabla}.\vec{j}_p = 0 \tag{39}
$$

with $\rho_p = \rho_{\partial t} + \rho_{\vec{\nabla}.} + \rho_{\vec{\nabla}\times}$. This explains why $\rho_{\partial t}$ and ρ_p have been differentiated in Eq. 36. In particular, note that the two partial charge densities $\rho_{\partial t}$ and $\rho_{\vec{\nabla}.}$ are non-zero for a propagating waves but their sum ρ_p vanishes as well as the corresponding current $\vec{j}_p$.

Yet, one faces at a difficulty: considering a constant electric charge, i.e. having no time dependence, the vector potential $\vec{A}_p$ has to increase linearly in time, which sounds unphysical. This is a direct consequence of the definition of the electric field $\vec{E}_p$ in Eq. 32 where the scalar potential V_p has been omitted. This issue will indeed be solved in section 3.3, since the electric charge is a physical quantity oscillating in time in the DuDy theory in contrast with the Standard Model [1, 6]. Nevertheless, we can already anticipate that the propagation equation is linked to the electromagnetic charges and currents. In contrast, the confinement equation (Eq. 31) has no real equivalent. It is therefore associated with new physics, linked here to the masses and related currents (remember that the confinement equation originates from the mass term in the energy momentum relation of the special relativity).

The analytical solutions of the propagation and confinement equations (Eqs. 30-31) are well-known [7]. They will be expressed here in spherical coordinates since the particle is assumed to be located at the referential origin as discussed in section 2.3:

$$
\frac{1}{4\pi k}\vec{\nabla}[z_l(kr)Y_l^m(\theta,\varphi)] = \frac{1}{4\pi kr}\left|\begin{array}{l}
Y_l^m(\theta,\varphi)r\dfrac{\partial}{\partial r}z_l(kr) \\
\dfrac{\partial}{\partial\theta}Y_l^m(\theta,\varphi)z_l(kr) \\
\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\varphi}Y_l^m(\theta,\varphi)z_l(kr)
\end{array}\right. \tag{40}
$$

$$
\frac{1}{4\pi}\vec{\nabla}\times[z_l(kr)Y_l^m(\theta,\varphi)\vec{r}] = \frac{1}{4\pi}\left|\begin{array}{l}
0 \\
\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\varphi}Y_l^m(\theta,\varphi)z_l(kr) \\
-\dfrac{\partial}{\partial\theta}Y_l^m(\theta,\varphi)z_l(kr)
\end{array}\right. \tag{41}
$$

$$\frac{1}{4\pi k}\vec{\nabla}\times\vec{\nabla}\times[z_l(kr)Y_l^m(\theta,\varphi)\vec{r}] = \frac{1}{4\pi kr}\left|\begin{array}{l} Y_l^m(\theta,\varphi)l(l+1)z_l(kr) \\ \dfrac{\partial}{\partial\theta}Y_l^m(\theta,\varphi)[krz_l(kr)]' \\ \dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\varphi}Y_l^m(\theta,\varphi)[krz_l(kr)]' \end{array}\right. \tag{42}$$

$Y_l^m(\theta,\varphi)$ are the modified Schmidt semi-normalized harmonics used in the magnetic community [8]:

$$Y_l^m(\theta,\varphi) = \sqrt{\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\varphi} \tag{43}$$

The function z_l appearing in Eqs. 40-42 is one of the spherical (modified) Bessel function. For the solutions of the propagation equation (Eq. 30), z_l corresponds to j_l and n_l whereas i_l and k_l will be used for the confinement equation (Eq. 31). They all fulfill the relation $[krz_l(kr)]' = \partial/\partial(kr)[krz_l(kr)] = [(l+1)z_l(kr) - krz_{l+1}(kr)]$.

Following the discussion of section 2.1, the wave-equations (Eqs. 30-31) driving the particle dynamics fulfill the criteria of general covariance. In other words, they are independent of the frame of reference and of the chosen coordinate transformation (Galilean or Lorentz). However, the choice made for $\vec{A}$ (Eqs. 40-42) gives a special role to the origin of the frame of reference (position of the particle) if one consider the diverging solutions $n_l(kr)$ and $k_l(kr)+i_l(kr)$ for $r \to 0$. Clearly, this kind of wave-functions breaks the translation invariance and completely modifies the topology for all eigenmodes due to their orthogonality. This is a key property in DuDy theory which will be further discussed later on: ***the presence of a particle do not modify the space-time entity as in general relativity [2]; it modifies all the allowed wave-functions***. This is indeed a rather intuitive and expected property.

Finally, I would like to briefly comment on the propagation and confinement Hamiltonians. In Cartesian coordinates, they can easily be deduced from the Hamiltonians of strings (see Eq. 4):

$$H_p = \int\left\{\frac{1}{2}\left(\frac{\partial\vec{A}_p}{\omega\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial\vec{A}_p}{k\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial\vec{A}_p}{k\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial\vec{A}_p}{k\partial z}\right)^2\right\}dV \tag{44}$$

$$H_c = \int\left\{\frac{1}{2}\left(\frac{\partial\vec{A}_c}{\omega\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\vec{A}_c}{k\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial\vec{A}_c}{k\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial\vec{A}_c}{k\partial z}\right)^2\right\}dV. \tag{45}$$

Written in this way, they also fulfill the general covariance discussed in section 2.1. If the propagation Hamiltonian has a rather standard form for longitudinal and transverse propagating waves, the confinement Hamiltonian is more surprising, but here again, it is obtained by exchanging $k \leftrightarrow ik$.

3.2 Fermions and bosons

As anticipated in sections 2.3 and 3.1, the present theory has to deal with a major difficulty: the vector potentials $\vec{A}_p$ and $\vec{A}_c$ describing the particles might contain diverging terms either at $r \to 0$ or at $r \to +\infty$. If the behavior at $r \to +\infty$ can be "regularized" by imposing a precise combination of $i_l(kr)$ and $k_l(kr)$ functions (see section 3.1), the diverging terms remains in most cases for $r \to 0$, where the particles are located. Intuitively, diverging terms indicate that particles cannot be brought too close to each other. This reminds a second issue raised in section 2.2: the Pauli exclusion principle as well as the distinction between fermions and bosons are no more accounted for by the definition of creation and annihilation operators (section 2.2). As we will discuss now, this two difficulties are closely related and will be solved conjointly.

The stability of a given particle is directly related to its ability to interact will all others. This is usually described by coupling Hamiltonians evaluating the overlap between particle wave-functions. Here, due to the vector nature of the wave-functions, they will involve terms like $\int \vec{A}_2.\vec{A}_1 dV$. None of them should diverge in order to ensure the particle stability. Hence, considering that $\vec{A}_2$ is a slowly varying function (see Fig. 2b) and writing $dV = r^2 dr \sin\theta d\theta d\phi$ in spherical coordinates, this implies that the vector potential $\vec{A}_1$ should not vary faster than $1/r^2$.

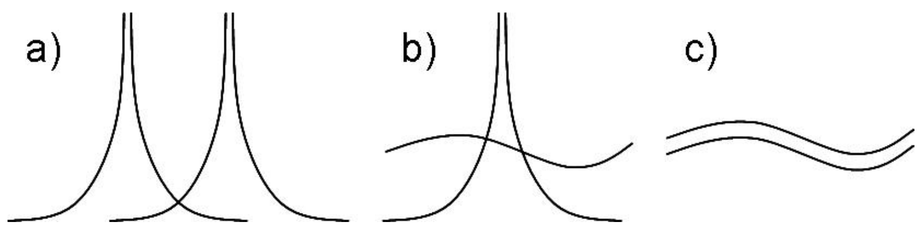

Figure 2: Schematic representation of particle interactions: "near field" fermion-fermion interaction (a), fermion-boson or "far-field" fermion-fermion interaction (b) and boson-bosons interaction (c). "Near field" and "far-field" have to be understood as in standard optics by comparing distances to the wavelength.

Applying the stability criterion $A \propto 1/r^l$ with $l \leq 2$ highly constrains the highest angular momentum l allowed for the solutions obtained in section 3.1. It also restricts the allowed linear combinations of spherical Bessel functions as discussed in section 2.3 (except for $j_l(kr)$ which is regular for all r). All "stable" solutions are finally reported in Table 1. For more clarity only the vector type ($\vec{\nabla}.$, $\vec{\nabla}\times$ and $\vec{\nabla} \times \vec{\nabla}\times$) and the spherical (modified) Bessel function are shown. The spherical harmonics $Y_l^m(\theta, \varphi)$ have been removed. The complete expression of the vector potentials can be easily recovered from Eqs. 40-42.

Angular momentum	*Stable particles*
$l = 0..\infty$ $l = 1..\infty$ $l = 1..\infty$	$\vec{\nabla}. \mid j_l$ $\vec{\nabla} \times \mid j_l$ $\vec{\nabla} \times \vec{\nabla} \times \mid j_l$
$l = 0$	$\frac{1}{2}\vec{\nabla}. \mid n_0$ $\frac{1}{2}\vec{\nabla}. \mid i_0 + k_0$
$l = 1$	$\frac{1}{2}\vec{\nabla} \times \mid n_1$ $\frac{1}{2}\vec{\nabla} \times \mid i_1 + k_1$ $\frac{1}{2}\vec{\nabla}. \mid n_1 - (i_1 + k_1)$ $\frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_1 - (i_1 + k_1)$ $\frac{1}{2}\vec{\nabla}. \mid (i_1 + k_1) + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_1$ $\frac{1}{2}\vec{\nabla}. \mid n_1 + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid (i_1 + k_1)$
$l = 2$	$\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_2$ $\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid i_2 + k_2$

Table 1: Stable particles fulfilling the $A \propto 1/r^l$ with $l \leq 2$ stability criterion.

A quick look at Table 1 shows that almost all remaining particles are either solutions of the propagation equation (Eq. 30) or solution of the confinement equation (Eq. 31). This is however not the case for the last solutions with $l = 1$ involving both n_l and $i_l + k_l$ diverging functions. This means that the propagation and confinement equations are not as independent as one might believe at first sight. Indeed, they will be strongly coupled in the present theory as already envisioned in the following discussion.

Since the creation and annihilation operators defined in section 2.2 do not any more distinguish fermions and bosons, this distinction must arise from the wave-functions. Invoking the Pauli exclusion principle, two identical fermions cannot occupy the same quantum state simultaneously. This would be precisely the case with diverging wave-functions if some interaction terms proportional to $\int \vec{A}_2\vec{A}_1 dV$ tends to infinity while the distance between the two particles is decreased (see Fig. 2a). In contrast, Bosons being particles which can be in the same quantum state, $\int \vec{A}^2 dV$ should not diverge imposing that A scales at most as $1/r$ for $r \to 0$. Hence, new definitions of fermions and bosons are postulated:

- ***fermions are particles described by vector potentials scaling as*** $1/r^2$ ***for*** $r \to 0$,
- ***bosons are particles described by vector potentials scaling as*** $1/r^l$ ***with*** $l \leq 1$ ***for*** $r \to 0$.

Applying these rules to the stable particles of Table 1 completely constrains the allowed bosons as shown in Table 2. The list of stable fermions has been voluntary reduced in order to retain linearly independent solutions. Note that all particles of Table 1 can be recovered by combining the fermions and bosons of Table 2.

Angular momentum	***Bosons***	***Fermions***
$l = 0..\infty$	$\vec{\nabla}. \mid j_l$	
$l = 1..\infty$	$\vec{\nabla} \times \mid j_l$	
$l = 1..\infty$	$\vec{\nabla} \times \vec{\nabla} \times \mid j_l$	
$l = 0$	$\frac{1}{2}\vec{\nabla}. \mid -n_0 + (i_0 + k_0)$	$\frac{1}{2}\vec{\nabla}. \mid n_0 + (i_0 + k_0)$
$l = 1$	$\frac{1}{2}\vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$	$\frac{1}{2}\vec{\nabla} \times \mid n_1 + (i_1 + k_1)$
	$\frac{1}{2}\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_1 + (i_1 + k_1)$	$\frac{1}{2}\vec{\nabla}. \mid -n_1 + (i_1 + k_1)$
	$\frac{1}{2}\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$	$\frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$
$l = 2$	$\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_2 + (i_2 + k_2)$	$\vec{\nabla}. + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_2 + (i_2 + k_2)$

Table 2: Bosons and linearly independent fermions allowed in DuDy theory.

Written as in Table 2, fermions and bosons have a singular property in the present theory: each fermions is associated to a given boson (the reversed property is not true). The symmetry between the corresponding expressions is striking, even in the case of mixed $\vec{\nabla}.$ and $\vec{\nabla} \times \vec{\nabla}\times$ vector potentials for $l = 1$. ***This clearly reminds the foundations of the SuSy theories*** [1]. But here, we have not postulated the existence of partners for each particle (the bosons involving only j_l functions have no symmetric fermion for instance). Bosons and fermions have naturally emerged while exploring the "stable" solutions of the two Hamiltonians describing dual dynamics.

It is also very interesting to note that, ***depending on the linear combination of propagation and confinement equation solutions, either fermions or bosons are obtained***. In order to illustrate this unique and crucial property, let us consider the $l = 0$ case where the two fermion vector potentials are $\vec{\nabla}. \mid n_0$ and $\vec{\nabla}. \mid (i_0 + k_0)$. If these solutions are added, $\vec{\nabla}. \mid n_0 + (i_0 + k_0)$ is also a fermion. However, if these solutions are subtracted, $\vec{\nabla}. \mid n_0 - (i_0 + k_0)$ has a boson character since the $1/r^2$ term cancels out. The same holds for all angular momenta l and can be easily verified by using the asymptotic expansions of the spherical (modified) Bessel functions.

I would like to stress here again that the postulated definition of fermions and bosons has drastically restricted the allowed angular momenta l and linear combinations of n_l, i_l and k_l functions. This is responsible for a kind of ***dynamical symmetry breaking in DuDy theory*** [1, 6]: the symmetry of the Hamiltonians driving the particle dynamics (Eq. 44 and 45) is partially lost in the allowed solutions. Note that this symmetry reduction deviates from the spontaneous symmetry breaking of the standard model [1, 6]: here, no additional boson is introduced in order to construct a specific potential. The physical argument is simply based on stability considerations requiring particles to have no vanishing life time due to diverging interaction integrals.

Finally, two types of bosons should be distinguished:

- ***"pure" bosons corresponding to intrinsically non diverging solutions*** (involving therefore only j_l functions),
- ***"composite" bosons obtained by combining two fermions*** (all other bosons in Table 2).

Composite bosons involve mixed n_l and $i_l + k_l$ contributions, i.e. they inherit physical properties from propagation and confinement equations (Eqs. 30-31), including a singularity at the origin ($r = 0$). They have therefore intrinsically a very different nature as compared to "pure" bosons, and will therefore father different physics.

3.3 Dual colored charges and magnetons

Once the vector potentials $\vec{A}$ describing the fermions have been "regularized" according to the previous section, they still scale as $1/r^2$. This specific r-dependence reminds the case of a punctual charge q in electrostatics

$$\Delta V = -\frac{q\delta(r)}{\epsilon_0} \;\rightarrow\; \vec{E} = -\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\vec{e_r} \tag{46}$$

where V and $\vec{E}$ are the scalar potential and the electric field associated with the punctual charge q. By analogy, one can anticipate that ***the*** $1/r^2$ ***dependence of the vector***

potentials in the DuDy theory is the signature of a point like object. In order to illustrate this discussion, let us first consider the fermions with $l = 0$ in Table 2. As discussed in the section 3.1, the equivalent in the present theory of the charge density appearing in Maxwell's equations is the partial charge density $\rho_{\partial t}$:

$$\rho_{\partial t} = \frac{1}{k}\vec{\nabla}.\left(-\frac{\partial}{\omega\partial t}\vec{A}_p\right) = \frac{1}{4\pi}Y_0^0(\theta,\varphi)n_0(kr).i\exp(i\omega t) \tag{47}$$

Hence, the longitudinal mode carry a charge density delocalized in space through the $n_0(kr)$ function. Applying now the divergence theorem, as usually done in electrostatics, reveals another contribution which has to be taken into account:

$$\begin{aligned}\rho_{\partial t} &= \left(\lim_{kr\to 0} 4\pi(kr)^2\vec{E}_p.\vec{e}_r\right).\delta(kr) \\ &= \left(-i\lim_{kr\to 0}(kr)^2Y_0^0(\theta,\varphi)\frac{\partial}{k\partial r}n_0(kr)\right).\delta(kr) \\ &= -Y_0^0(\theta,\varphi)\delta(kr).i\exp(i\omega t) = q_{\partial t}\delta(kr).\end{aligned} \tag{48}$$

It corresponds to a point charge $q_{\partial t} = \lim_{kr\to 0} 4\pi(kr)^2\vec{E}_p.\vec{e}_r$ oscillating in time. This is not an expected feature in the standard model [1, 6], where the electric charges are constants. One might argue that the time dependance is separately encoded in the wave-function in the standard model, leading indeed to the same physics. This will be analyzed in more detail in section 3.5 where the specific role of the time dependent functions is discussed. Nevertheless, as seen in Eqs. 47 and 48, these oscillating point charges, characterized by Dirac delta functions, are accompanied by "surrounding clouds" at larger distances, i.e. by delocalized waves. This fundamental property will be discussed in section 3.4.

Finally, applying the gradient operator to Eqs. 47 and 48

$$\left(\frac{\partial^2}{\omega^2\partial t^2} - \frac{1}{k^2}\Delta\right)\vec{A}_p = -\frac{1}{k}\vec{\nabla}\left[Y_0^0(\theta,\varphi)\exp(\pm i\omega t)\delta(kr)\right] = \vec{j}_p \tag{49}$$

an important property comes out: ***a source term (charge current) $\vec{j}_p$ associated with a point-like particle has to be incorporated in the propagation equation***.

Exactly the same approach can be applied to the confinement equation (Eq. 31) associated here with the mass

$$m_{\partial t} = \lim_{kr\to 0} 4\pi(kr)^2\vec{E}_c.\vec{e}_r \tag{50}$$

by replacing $n_0(kr)$ by $i_0(kr) + k_0(kr)$. This leads to a modified confinement equation including current sources as well:

$$\left(\frac{\partial^2}{\omega^2\partial t^2} + \frac{1}{k^2}\Delta\right)\vec{A}_c = \frac{1}{k}\vec{\nabla}\left[Y_0^0(\theta,\varphi)\exp(\pm i\omega t)\delta(kr)\right] = \vec{j}_c. \tag{51}$$

The above discussion on the introduction of source terms is slightly more difficult for fermions having $l > 0$. ***The $1/r^2$ dependence suggests to use the concept of point charge and mass whatever the value of the angular momentum l. However, owing to the presence of spherical harmonics these extended definitions of charges and masses in DuDy theory differ from the usual ones by a "color" corresponding to the value of the azimuthal angular momentum $\mathbf{m}$ for $\mathbf{l} > \mathbf{0}$***. Note that the $1/r^2$ dependence of the vector potentials has often been obtained by using linear combinations of $n_l(kr)$ and $i_l(kr) + k_l(kr)$. It finds its origin in both propagation and confinement equations (Eqs. 30 and 31). Hence, ***mass, charge and colors are unified in the DuDy theory by defining dual colored charges as***

$$q_l^m = \frac{\lim_{kr\to 0}\left(\int 4\pi(kr)^2\vec{E}.\vec{e}_r.Y_l^{m*}(\theta,\varphi)\sin\theta d\theta d\varphi\right)}{\int Y_l^{m*}(\theta,\varphi)Y_l^m(\theta,\varphi)\sin\theta d\theta d\varphi}Y_l^m(\theta,\varphi) \tag{52}$$

where $\vec{E} = \vec{E}_p + \vec{E}_c$. The normalizing term is necessary due to the specific definition of spherical harmonics used in the present work (see Eq. 43). For $l = 0$ the standard charge is clearly recovered since $Y_0^0(\theta,\varphi) = 1$. Using the asymptotic developments of the $n_l(kr)$ and $i_l(kr) + k_l(kr)$ functions [7], all fermions are found to have the same kr dependence up to a sign: $\pm 1/(kr)^2$. Hence, the expression of the dual charges reduces to

$$q_l^m = (-1)^{l+1}Y_l^m(\theta,\varphi).i\exp(i\omega t) \tag{53}$$

so that, apart from the temporal phase term, the colored charges correspond to the spherical harmonics $Y_l^m(\theta,\varphi)$ multiplied by $(-1)^{l+1}$.

It is important to emphasize that ***the definition of dual colored charges assigns charges only to longitudinal $\vec{\nabla}.$ vector potentials***. The transverse modes $\vec{\nabla}\times$ and $\vec{\nabla}\times\vec{\nabla}\times$ are charge-less but have instead a rotational component. Following a similar approach, the notion of dual colored magnetons μ_l^m can be introduced. Both charges and magnetons lead to source terms that are harmonic in time and therefore can be considered as vibrating modes. For $l = 1$ for example, a dual colored charge is schematically described by an oscillating dipole composed of two charges linked by a current (cf. Fig. 3). In contrast, the source generated by a dual colored magneton can be represented by a small loop with an oscillating current. Reducing their dimensions to zero gives the right picture of the source terms. ***In that sense DuDy theory might have some connections with string theories where the extra-dimensions corresponding to the oscillating sources (opened and closed loops) are compacified to give point-like object***. More precisely, the dipole can be viewed as an open string that can be transformed into the loop (closed string) by linking the two charges, and vice-versa. The number of extra-dimensions would be given by the final number of particles present in the DuDy theory.

Finally, I would like to emphasize a very specific property related to dual charges and magnetons. As discussed in the previous section, the composite bosons of Table 2

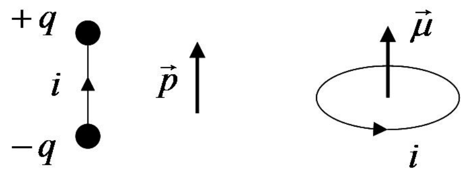

Figure 3: Schematic representation of dual charges and magnetons.

are obtained by combining two fermions. Therefore, ***even if composite bosons have no charge nor magneton, they do have source terms corresponding to the two fermions involved. In that sense, they do not behave as pure bosons, which have no singularity and therefore no source term in their definition***.

3.4 Wave-source duality

One of the main message of the previous section is that a particle is described by a dual colored charge or magneton surrounded by a delocalized cloud. Generalizing Eq. 47 for any (l, m) values gives

$$\begin{aligned}
\rho_{\partial t} &= \frac{1}{k}\vec{\nabla}.\left(-\frac{\partial}{\omega\partial t}\vec{A}_p\right) \\
&= \frac{i}{4\pi}Y_l^m(\theta,\varphi)n_l(kr)\exp(i\omega t) - i.Y_l^m(\theta,\varphi)\delta(kr)\exp(i\omega t) \\
\rho_{\partial t} &= q_l^m\left[\delta(kr) - \frac{n_l(kr)}{4\pi}\right] \qquad (54)
\end{aligned}$$

Hence, in the present theory, ***the eigenmodes have both particle*** $\delta(kr)$ ***and wave*** $\frac{n_l(kr)}{4\pi}$ ***components offering a natural framework to investigate the wave-particle duality [6] or more precisely the "wave-source" duality***. In quantum field theories, this duality is accounted for by a field operator acting on particle states [6]. Here, it is directly encoded into the vector potential (wave) having a singularity at the origin (source). Note that this deviates from electrostatics, where the divergence of the electric field is zero outside the point charge: the divergence of the electric field spread out in the entire space with an oscillating amplitude proportional to the point charge. This explains why an electron, viewed as a point source via the $\delta(kr)$ function, experiences diffraction, via the $n_l(kr)$ function, while passing through a slit. In other words, ***the source creates the field but the field tells to the source how to move***. This has many implications that will be analyzed later, but the one of concern at present is related the creation and annihilation operators. The fact that source terms generate the fields imposes the following postulate:

- ***In the DuDy theory, the creation and annihilation operators act directly on the source terms and not on the waves (vector potentials).***

This postulate is indeed rather intuitive and solves unwanted features related to causality: the operators create/annihilate locally the source and then the surrounding fields builds up in time. With this, the field is not instantaneously modified at large distances, which otherwise sounds unphysical. This postulate translates the propagation and annihilation equations (Eqs. 30 and 31) into the following form:

$$\begin{aligned}\left(\frac{\partial^2}{\omega^2\partial t^2}-\frac{1}{k^2}\Delta\right)\vec{A}_p &= \frac{1}{k}\vec{\nabla}\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\delta(kr)\right].\left(p^{\dagger}_{\vec{\nabla}.{}_l^m}(t)+p_{\vec{\nabla}.{}_l^m}(t)\right) \\ &+ \vec{\nabla}\times\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\vec{r}\delta(kr)\right].\left(p^{\dagger}_{\vec{\nabla}\times{}_l^m}(t)+p_{\vec{\nabla}\times{}_l^m}(t)\right) \\ &+ \frac{1}{k}\vec{\nabla}\times\vec{\nabla}\times\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\vec{r}\delta(kr)\right].\left(p^{\dagger}_{\vec{\nabla}\times\vec{\nabla}\times{}_l^m}(t)+p_{\vec{\nabla}\times\vec{\nabla}\times{}_l^m}(t)\right)\end{aligned} \tag{55}$$

$$\begin{aligned}\left(\frac{\partial^2}{\omega^2\partial t^2}+\frac{1}{k^2}\Delta\right)\vec{A}_c &= \frac{1}{k}\vec{\nabla}\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\delta(kr)\right].\left(c^{\dagger}_{\vec{\nabla}.{}_l^m}(t)+c_{\vec{\nabla}.{}_l^m}(t)\right) \\ &+ \vec{\nabla}\times\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\vec{r}\delta(kr)\right].\left(c^{\dagger}_{\vec{\nabla}\times{}_l^m}(t)+c_{\vec{\nabla}\times{}_l^m}(t)\right) \\ &+ \frac{1}{k}\vec{\nabla}\times\vec{\nabla}\times\left[Y_l^m(\theta,\varphi)\exp(i\omega t)\vec{r}\delta(kr)\right].\left(c^{\dagger}_{\vec{\nabla}\times\vec{\nabla}\times{}_l^m}(t)+c_{\vec{\nabla}\times\vec{\nabla}\times{}_l^m}(t)\right).\end{aligned} \tag{56}$$

In the source terms, creation and annihilation operators have been introduced with the following convention: p and c relates to propagation (Eq. 55) and confinement (Eq. 56) equations; $\vec{\nabla}.$, $\vec{\nabla}\times$ and $\vec{\nabla}\times\vec{\nabla}\times$ indicates the type of source term and (l,m) its color. Important properties comes out:

- first, ***pure bosons as defined in section 3.2 and Table 2 do not have source terms and therefore have neither creation/annihilator operators nor definite positions (no*** $\delta(kr)$ ***function). It that sense, they have no definite quantum property.***
- second, ***composite fermions as defined in section 3.2 and Table 2 have been assigned to a pair of creation/annihilator operators for their propagation and confinement source terms***.
- third, ***composite bosons as defined in section 3.2 and table 2 have also paired creation/annihilator operators***. They have therefore "intermediate" properties between fermions and pure bosons, i.e. no dual charge nor magneton but definite positions arriving from the $\delta(kr)$ functions.
- fourth, the resolution of the propagation (Eq. 55) and confinement (Eq. 56) equations always involves a particular and a general solution. The particular solution corresponds to the solution of the equation with the source term (composite bosons or fermions), whereas the general solution is obtained without the source term

(pure bosons). Initial conditions fix the amplitude of the general solution as it is the case for a classical mechanical oscillator. Hence, ***the quantum properties of the pure bosons are inherited from that of the composite bosons or fermions***.

Combining all source terms into the generic source currents $\vec{j}_p$ and $\vec{j}_c$, the propagation and confinement Hamiltonians (Eqs. 44 and 45) write:

$$\begin{aligned} H_p &= \int \left\{ \frac{1}{2}\left(\frac{\partial \vec{A}_p}{\omega \partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_p}{k \partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_p}{k \partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_p}{k \partial z}\right)^2 \right\} dV \\ &+ \int \vec{j}_p.\vec{A}_p dV \qquad (57) \\ H_c &= \int \left\{ \frac{1}{2}\left(\frac{\partial \vec{A}_c}{\omega \partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_c}{k \partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_c}{k \partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_c}{k \partial z}\right)^2 \right\} dV \\ &+ \int \vec{j}_c.\vec{A}_c dV \qquad (58) \end{aligned}$$

Due to the $\vec{j}_p.\vec{A}_p$ and $\vec{j}_c.\vec{A}_c$ production terms, the source currents $\vec{j}_p$ and $\vec{j}_c$ have to create the waves $\vec{A}_p$ and $\vec{A}_c$ in order to reach one of the stable particle state shown in table 2. Symmetrically, ***the annihilation of a given particle removes the source terms and the remaining waves have to evolve toward pure bosons since no more source is present***.

There is a final fundamental consequence of the wave-source duality in the DuDy theory: the notion of locality. It is naturally encoded in the sources due to the $\delta(kr)$ contribution. But as we have seen, both source and wave components have to be considered. Modifying at a given position the wave will affect the particle, but this interaction will not be instantaneous. The wave perturbation has first to propagate toward the particle location before acting on it. Hence, ***the only source of non-locality in the DuDy theory is provided by symmetry, which by nature can be non-local***.

3.5 Spin and anti-particles

If the criteria established in section 3.2 has allowed differentiating the bosons and fermions of the DuDy theory, it does not explain why two fermions having opposite spins do not interact, i.e. why $\int \vec{A}_2 \vec{A}_1 dV = 0$. If one consider the simplest fermion in Table 2 corresponding to $l = 0$, the only possibility to have orthogonal vector potentials is to consider that the spin is associated with the time dependence of $\vec{A}$. As a matter of fact there are two orthogonal time-dependent solutions of propagation and confinement equations (Eq. 30 and 31):

$$\exp(\pm i\omega t) \qquad (59)$$

or their linear combinations leading to sine and cosine functions. Therefore, ***the two orthogonal time-dependent solutions*** $\exp(\pm i\omega t)$ ***will be associated with the two orthogonal quantum states having opposite spins***, in contrast with what is usually done in field theories. The use of negative frequencies in DuDy theory is compatible with the definitions of Hamiltonians (cf. Eqs 57 and 58) since the $\omega \to -\omega$ transformation no longer leads to negative energies (remember that $H \neq i\hbar\frac{\partial}{\partial t}$ as discussed in sections 2.1 and 2.2).

With this definition, the notion of spin finds a natural interpretation. As we have seen in section 3.3, dual colored charges are proportional to

$$q_l^m = \mp i Y_l^m(\theta,\varphi)\exp(\pm i\omega t) \propto \exp(im\varphi \pm i\omega t) \tag{60}$$

where the $\mp$ and $\pm$ signs have been introduced. Searching the positions having a constant phase imposes $\varphi(t) = \mp\omega t/m + cte$, i.e. a spinning effect depending of the sign in front of ω. This is precisely the intuitive conception of spins as given by a rotation around the z-axis. Note however that this colorful interpretation fails in figuring out the nature of spin for $l = 0$: in this case, $m = 0$ implies no rotation and therefore no classical equivalent of the spinning effect.

One fundamental property can be further deduced from Eq. 60: the vector potentials with (l, m, ω) and $(l, -m, -\omega)$ lead to the same spatio-temporal behavior (if $m \neq 0$). More precisely, the solutions are orthogonal in space and time and imposing the phase to be constant leads to the same spinning effect. This means that despite their orthogonality (implying distinct particles), the solutions share many physical properties. This provides a tentative definition for the anti-particles as having:

- the same solution type ($\vec{\nabla}.$,$\vec{\nabla}\times$ or $\vec{\nabla}\times\vec{\nabla}\times$),
- the same angular momentum l,
- the opposite azimuthal angular momentum $m \leftrightarrow -m$,
- the opposite spin $\exp(i\omega t) \leftrightarrow \exp(-i\omega t)$,
- the opposite charge (π phase shift)

with respect to the particle, in close connection with the charge-parity-time (CPT) symmetry [6]. In order to test this definition, let us consider the annihilation of an electron-positron pair [6]:

$$e^+ \quad + \quad e^- \quad \to \quad \gamma \quad + \quad \gamma \tag{61}$$

In view of the DuDy theory, the dual colored charge has to be conserved during the annihilation process. Photons being charge-less, the dual colored charges of the electron and the positron must cancel out. In other words, they must have opposite signs. This is indeed not compatible with the fact that they have opposite spins (remember that the time dependence enters in the definition of charges, cf. section 3.3).

As a consequence, ***the relation between particles and anti-particles has to be reconsidered in the DuDy theory. More precisely, it is postulated that***

- ***particles and anti-particles simply have opposite dual charges or magnetons (π phase shift)***

meaning that they have the same solution type ($\vec{\nabla}.$,$\vec{\nabla}\times$ or $\vec{\nabla}\times\vec{\nabla}\times$), the same angular momentum l, the same azimuthal angular momentum m, the same spin $\exp(\pm i\omega t)$, but π phase shift. In order to avoid confusion with the positron of the Standard Model[1], the anti-particle associated with the electron in the DuDy theory will be called δ-positron (the same applies to all anti-particles). The dual colored charges of the electron and the δ-positron being now exactly opposite, the source terms cancel out during the annihilation process. However, the wave accompanying the dual colored charges do remain. Hence, despite the negative mass and positive charge of the δ-positron, there is no "nullification" issue [9] in the present theory. Since there is no more source term, the spatial distribution of the waves evolves toward that of pure bosons, as discussed in section 3.4, allowing to fulfill the energy conservation. As a consequence, the γ rays appearing in Eq. 61 are directly linked to the pure bosons of Table 2, as might have been anticipated since a while. It is therefore time to make the connection between DuDy and Standard Model particles and to establish the particle interactions.

3.6 Fundamental interactions and elementary particles

Four fundamental interactions are generally accepted as describing the behavior of Nature [1, 6] : the gravitational, electromagnetic, strong nuclear and weak nuclear interactions. All are mediated by bosons, or hypothesized as such, since the gravitons have not been observed yet. In view of sections 3.2 and 3.4 describing fermions, bosons and the wave-source duality, the possibilities are more diverse in the present theory. More precisely, two kinds of interactions can be envisioned:

- ***"indirect" interactions where the fermion-fermion interactions is mediated by bosons***, as in the Standard Model [1, 6],
- ***"direct" fermion-fermion interactions involving a coupling through their wave components or between the dual colored source of a given fermion and the wave of the other fermion***.

Direct interactions are clearly a new feature in the present theory compared to the Standard Model. The specific vector potential expressions given in Table 2 are therefore key ingredients to disentangle all contributions.

3.6.1 Electromagnetic and gravitational interactions

As discussed in section 3.3, mass and charge are unified in the present theory leading to dual colored charges. Electromagnetic and gravitational interactions have therefore to be treated on an equal footing. For this proposal, gravitons and photons, which are bosons carrying no charge and no mass [1], will be described by the first three pure bosons of Table 2. Thanks to their $\vec{\nabla}.$ and $\vec{\nabla}\times$, $\vec{\nabla}\times\vec{\nabla}\times$ generating operators, and due to the fact that photons sustains transverse waves with two crossed polarizations, the following identification is proposed:

- ***δ-gravitons are the longitudinal modes*** $\vec{\nabla}. \mid j_l$,
- ***δ-photons are the transverse modes*** $\vec{\nabla}\times \ \mid j_l$ ***and*** $\vec{\nabla}\times\vec{\nabla}\times \ \mid j_l$,

where "δ-" refers to "DuDy" (as far as the equivalence between the particles of the Standard Model and the present theory has not been fully demonstrated, the prefix δ will be employed). Note that this assignment of pure bosons is compatible with the separation into symmetric (gravitons) and antisymmetric (photons) parts [2].

δ-photons and δ-gravitons involving j_l functions only, they are mass-less and charge-less bosons mediating interactions with an infinite range, as required. The j_l functions being orthogonal to n_l functions, ***δ-photons cannot interact linearly with the wave component $\vec{A}_p$ of the composite bosons and fermions. However, they do interact with the source terms through indirect interactions***. This coupling scheme is the analogous to the standard electromagnetic interaction of the Standard Model [1, 6]. In contrast, $i_l + k_l$ functions are not orthogonal to the j_l functions, meaning that ***the δ-gravitons do not interact solely with the source terms through indirect interactions; they also interact with the wave component of the particle states. This results in an effective mass, which vary from particle to particle in contrast with the charge, which is the same up to sign***.

The previous gravitational and electromagnetic interactions have been expressed in terms of indirect interactions (i.e. interactions mediated by bosons). There is however the other type of interactions corresponding to direct interactions and therefore involving the particle waves. As we have shown in section 3.2, ***the regularized vector potentials describing fermions scale as $1/r^2$ for $r \to 0$. This provide an additional force having the right r-dependence***. More precisely, gravitational and electrostatic forces $\vec{F}$ derive from potential energies U through the well-known relation $\vec{F} = -\vec{\nabla}U$:

$$U = -G\frac{m_1 m_2}{r} \quad for\ Newton\ forces \tag{62}$$

$$U = \frac{1}{4\pi\epsilon_0}.\frac{q_1 q_2}{r} \quad for\ Coulomb\ forces \tag{63}$$

The sign change between gravitational and electrostatic potential energies is in agreement with the sign changes of the potential terms in the propagation and confinement equations (Eqs. 55-56), explaining why masses attract each-other in contrast with charges having the same sign. Note that this sign change in the forces and potential energies naturally appeared while making the $k \leftrightarrow ik$ transformation of the driving equations (Eqs. 30-31). Newton and Coulomb forces are therefore dual forces in the present theory. In addition, mass and charges being merged in dual colored charges for particles with $l > 0$, the indirect interactions describing Newton and Coulomb forces are unified in the DuDy theory. This is indeed not really an expected feature in view of the hierarchy problem [1], since the gravitational force is at least 32 orders of magnitude smaller than the electromagnetic interaction. There is however a fundamental difference between direct and indirect interactions which has not been discussed yet. If both types of interactions are proportional to the dual colored charges,

- direct interactions depends only on the relative positions of the two particles (1 and 2). They are of strong intensity when the source 1 enters in the near-field ($kr \to 0$) of the wave 2 and vice-versa.

- indirect interactions are driven by the occupation numbers of bosons at all frequencies. Owing to the definition of the creation and annihilation operators postulated in the DuDy theory (cf. section 2.2), this interaction strictly vanishes in the absence of dual photons (the expectation value for the fields is strict zero for zero occupation numbers).

Gathering all aspects discussed above, ***gravitational and electromagnetic interactions can be unified into dual electromagnetic forces, where the direct interaction plays the role of the Coulomb forces of the Standard Model and indirect interaction replaces the gravitational force. If correct, this dual electromagnetic interaction in DuDy theory is likely to solve the hierarchy problem*** [1]. More precisely, at an early stage of universe, when the dual photon bath was very dense, gravitation and coulomb forces might have been of the same order of magnitude, but nowadays the remaining cosmological microwave background [1] is not sufficient to provide a strong gravitational force, explaining the huge difference between these two interactions. Following this idea, ***the effective gravitational constant [1] has necessarily evolved in the present model along with the universe history down to its actual value***.

Last but not least is the question of the equivalence principle between gravitational and inertial masses [10]. Many experiments have demonstrated the perfect equality between these two entities. However it is surprising that, despite the hierarchy problem [1] stating that gravitation is at least 32 orders of magnitude weaker than other forces, the same mass governs the time evolution through the Newton's laws. More precisely, if inertial and gravitational masses have the same origin (and value) they express themselves with a completely different strength: the mass vanishingly contributes to the forces but strongly imposes it's inertia, leading to an additional type of hierarchy problem. An interpretation of this puzzling effect is offered in the present theory by the difference between direct and indirect interaction schemes combined with wave-particle duality:

- ***the long range gravitational interaction is due to the indirect coupling between the pure bosons of the propagation equation (Eq. 55) and the source of the confinement equation (Eq. 56)***. It is therefore of tiny intensity, as discussed above.
- ***when accelerated, the point source interacts with its own wave***. This corresponds to a direct interaction and has therefore a large magnitude. To give a clear picture, the point source has to push/pull its own cloud explaining the notion of inertia.

Hence, in the DuDy theory, the role of the inertial mass is not linked to a deformation of the space-time entity [2], to an hypothetical Ether or to a quantum vacuum background. It directly originates from the specific wave-source duality introduced in section 3.4 and from the two different interaction types resulting from the definition of bosons and fermions in section 3.2.

3.6.2 Strong interactions

The strong interaction is known to confine quarks into composite particles [1, 6]. As suggested by its name this interaction will originate in the present theory from the confinement equation (Eq. 56). More precisely, it is associated with the i_l and k_l functions which have been found to be always associated in Table 2. The i_l functions being regular for $r \to 0$ [7], the corresponding particles might be considered as bosons having neither electric charge nor mass but two spin states associated with $\exp(\pm i\omega t)$. ***This asymptotic behavior leading to small amplitudes for*** $r \to 0$ ***reminds the asymptotic freedom of quarks into hadrons*** [6]. On the other hand, owing to their diverging asymptotic expansion given by Eq. 28 for $r \to +\infty$, they are necessary linked to the k_l function: as their amplitude grows exponentially from the origin, the energy needed to separate them will grow exponentially with their distance. ***This leads to a type of confinement [1, 6]. Hence, the*** i_l ***functions will be called the*** δ***-gluons of the DuDy theory***. The asymptotic divergence of the corresponding interaction (since $i_l \to \infty$ for $r \to \infty$) is not present in the Standard Model [1, 6] and will be discussed in section 4.2.3.

3.6.3 Weak interactions

Once the i_l and k_l functions have been gathered, the resulting asymptotic expansion of the function $i_l + k_l$ reads [7]

$$i_l(kr) + k_l(kr) \approx \frac{e^{-kr}}{kr} \quad \text{for} \quad kr >> l(l+1)/2. \tag{64}$$

The corresponding passive interactions have a short range owing to the exponential decrease. The special form of Eq. 64 clearly reminds the weak interactions [1, 6] and more precisely the Yukawa potential [1, 6], arising from the exchange of massive scalar fields between two fermions in the Standard Model. In the present case, i_l and k_l functions are solution of the confinement equation which is directly linked to the notion of mass (cf. section 2.3).

Apart from the charge-less and mass-less pure bosons corresponding to δ-photons and δ-gravitons, all other composite fermions and bosons of Table 2 carry dual colored charges or magnetons and are subject to weak interactions. In order to evidence additional properties, one has to go back to the definition of the $\vec{\nabla}.$, $\vec{\nabla}\times$ and $\vec{\nabla}\times\vec{\nabla}\times$ operators given in Eqs. 40-42:

- due to its components, the vector potential $\frac{1}{4\pi k}\vec{\nabla}\times[z_l(kr)Y_l^m(\theta,\varphi)\vec{r}]$ is locally and globally orthogonal to the two other vector potentials. Furthermore, having no longitudinal component, it has neither charge nor mass.
- $\frac{1}{4\pi k}\vec{\nabla}[z_l(kr)Y_l^m(\theta,\varphi)]$ and $\frac{1}{4\pi k}\vec{\nabla}\times\vec{\nabla}\times[z_l(kr)Y_l^m(\theta,\varphi)\vec{r}]$ have the same angular dependence. The difference between these two vectors, leading to a longitudinal and a transverse character, resides only in their radial dependence. Hence, these solutions are globally orthogonal but not locally orthogonal.

Adding the fact that for $l=1$ there are three colors: $m=[-1,0,1]$, ***the $\vec{\nabla}.$ and $\vec{\nabla}\times\vec{\nabla}\times$ vector potentials are likely to play the role of the up and down quarks of the Standard Model*** [6, 1]. This assignment is further suggested by the fact that the symmetry between up and down quarks is broken in the Standard Model (they have different masses). Here, this symmetry breaking is directly encoded in their different vector potentials. There is however a fundamental difference between the Standard-Model [1, 6] and the DuDy theory: ***the δ-quarks do not have fractional electric charges ($\pm 1/3$ or $\pm 2/3$) as in the Standard Model but either dual colored charges for the longitudinal solution $\vec{\nabla}.$ or zero charge for the transverse one $\vec{\nabla}\times\vec{\nabla}\times$*** (see Eq. 53).

Thanks to the specific formulas used in the present theory for the spherical harmonics (see Eq. 43), the following relation is fulfilled:

$$\sum_{-l\leq m\leq l} |Y_l^m(\theta,\phi)|^2 = 1 \tag{65}$$

Hence, defining the charge of a composite particle by

$$q = \sqrt{\sum |q_l^m|^2} \tag{66}$$

one obtains for composite particles holding three colored quarks:

- $q = \sqrt{\sum_{-l\leq m\leq l} |q_l^m|^2} = 1$ for the longitudinal $\vec{\nabla}.$ vector potential, as expected for protons. The associated particles will therefore be called the δ-proton quarks.
- $q = \sqrt{\sum_{-l\leq m\leq l} |q_l^m|^2} = 0$ for the transverse $\vec{\nabla}\times\vec{\nabla}\times$ vector potential, as expected for neutrons. The associated elementary particles carry however an intrinsic magnetic moment given by the dual colored magneton μ_l^m, and will be named δ-neutron quarks accordingly.

Concerning the $\vec{\nabla}\times$ vector potential, we have seen that it has neither charge nor mass and is locally orthogonal to all other fermions and bosons with the exception of the $\vec{\nabla}\times$ boson. It is thus expected to hardly interact with matter. For all these reasons, ***the fermion $\vec{\nabla}\times$ vector potential will be referred as the δ-neutrino quarks in the DuDy theory***. These quarks have two interesting properties:

- the δ-neutrino quarks are generated by the $\vec{\nabla}\times$ transverse operator only and not by the other transverse operator $\vec{\nabla}\times\vec{\nabla}\times$. As a consequence, the symmetry between the two orthogonal polarizations of transverse modes is broken. This might explain why only left-handed chirality has ever been observed for the neutrinos of the Standard Model [1].
- as for the δ-proton and neutron quarks, δ-neutrino quarks are $l = 1$ particles and therefore exist in thee colors $m = -1; 0; +1$, in contrast with the neutrinos of the Standard Model [1].

The $l = 1$ composite bosons can now be labeled by analogy with the bosons of the Standard Model [1, 6] and in view of the charge content of the fermions entering in their definition:

- $\frac{1}{2}\tilde{\nabla}\times \mid \mathbf{n_1} - (\mathbf{i_1} + \mathbf{k_1})$ ***is the*** δ-$\mathbf{Z^0}$ ***boson***,
- $\frac{1}{2}\tilde{\nabla}. + \frac{1}{2}\tilde{\nabla}\times\tilde{\nabla}\times \mid \mathbf{n_1} + (\mathbf{i_1} + \mathbf{k_1})$ ***is the*** δ-$\mathbf{W^-}$ ***boson***,
- $\frac{1}{2}\tilde{\nabla}. + \frac{1}{2}\tilde{\nabla}\times\tilde{\nabla}\times \mid -\mathbf{n_1} + (\mathbf{i_1} + \mathbf{k_1})$ ***is the*** δ-$\mathbf{W^+}$ ***boson***.

The remaining fermions to be identified are associated with $l = 0$ and $l = 2$ angular momenta:

- for $l = 0$, the fermion has only one color ($m = 0$). Its charge has a unity amplitude and a reversed sign (before the n_0 function) compared to the δ-proton quarks. Hence, ***the*** $\boldsymbol{\frac{1}{2}\tilde{\nabla}. \mid \mathbf{n_0} + (\mathbf{i_0} + \mathbf{k_0})}$ ***vector potential is named the δ-electrons***, in analogy with the electrons of the Standard Model,

- ***for*** $\mathbf{l = 5}$***, the fermions have 5 colors and experience an extended type of weak and strong interaction. They will be called δ-technicolor quarks, in reference to technicolor theories [1]***.

The last but not least particle is the composite boson with $l = 0$. This purely radial vector potential has no dual colored charge, no spinning effect ($m = 0$) and does not directly interact with the δ-photons and δ-neutrino quarks, which are the only purely transverse particles in Table 3. For all these reasons, ***the*** $\boldsymbol{\frac{1}{2}\tilde{\nabla}. \mid -\mathbf{n_0} + (\mathbf{i_0} + \mathbf{k_0})}$ ***vector potential share many properties with the Higgs boson of the Standard Model [1] although the latter is a scalar field. It will be called the δ-Higgs or δ-X boson***, so that all fermions and bosons of the DuDy theory can now be listed in Table 3.

To conclude this part, let us see if the present theory reproduce some of the unique properties of the weak interaction of the Standard Model [1, 6]:

- first, it is the only interaction capable of changing the flavor of quarks (i.e., of changing one type of quark into another). This is clearly the case if the δ-proton and neutron quarks interact with the δ-W^+ and δ-W^- (see Table 3).

- second, it is the only interaction which violates parity-symmetry [1, 6]. This is also true in the present theory since, as discussed in section 2.3, $i_l + k_l$ functions has no definite parity in contrast with all other spherical (modified) Bessel functions taken separately (j_l, n_l, i_l and k_l).

- third, it is propagated by carrier particles (known as gauge bosons) that have significant masses. As discussed in section 3.3, the composite bosons have no dual colored charges in the sense that they have only a $1/r$ dependance. However, they have been obtained by combining fermions that do hold dual colored charges or magnetons.

Angular momentum	***Bosons***	***Fermions***
$l = 0..\infty$	δ-graviton $\vec{\nabla} \mid j_l$	
$l = 1..\infty$ $l = 1..\infty$	δ-photons $\vec{\nabla} \times \mid j_l$ $\vec{\nabla} \times \vec{\nabla} \times \mid j_l$	
$l = 0$	δ-Higgs or δ-X $\frac{1}{2}\vec{\nabla} \mid -n_0 + (i_0 + k_0)$	δ-electron quark $\frac{1}{2}\vec{\nabla} \mid n_0 + (i_0 + k_0)$
$l = 1$	δ-Z^0 $\frac{1}{2}\vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$ δ-W^- and δ-W^+ $\frac{1}{2}\vec{\nabla} + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_1 + (i_1 + k_1)$ $\frac{1}{2}\vec{\nabla} + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$	δ-neutrino quarks $\frac{1}{2}\vec{\nabla} \times \mid n_1 + (i_1 + k_1)$ δ-proton and δ-neutron quarks $\frac{1}{2}\vec{\nabla} \mid -n_1 + (i_1 + k_1)$ $\frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$
$l = 2$	δ-Y $\vec{\nabla} + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_2 + (i_2 + k_2)$	δ-technicolor quarks $\vec{\nabla} + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_2 + (i_2 + k_2)$

Table 3: Summary of the elementary particles in DuDy theory.

4 DuDy and the generations of composite particles

4.1 Nonlinear Lagrangians and wave-equations

The elementary particles of the DuDy theory have been listed in Table 3, suggesting connections with the elementary particles of the Standard Model [1]. This classification opens indeed many questions: what is the origin of the three (or more) generations of particles? Why do the generations share exactly the same quantum numbers but have different masses? Or in other words, why masses are (so) different from generation to generation and not the charges? These are clearly missing features in the present theory. New physics has to be included.

The three generations are associated with three different masses. Three masses means three energies and three energies imply quantification. Clearly, the continuum of states (the wave-number k is not fixed) has to be quantized, implying the introduction of a new potential, i.e. to a new coupling scheme between particles (or even to self coupling). Therefore, ***the wave-equations will no longer be linear but become non-linear***. The choice of this coupling potential is highly constrained as it has to be compatible with the previous discussion on particles and interactions, at least down to some small length scales so that the distinction between fermions and bosons still holds at low energies. The only clear rule is that the potential energy must involve scalar products of the vector potentials since the Lagrangian densities are scalar quantities. After several trials, ***the following coupling Hamiltonian is postulated***

$$U = \left[\sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]^2 - \sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 \tag{67}$$

for particles staying at the same position, here the origin of the spherical coordinates. $\vec{A}_p^i$ and $\vec{A}_c^i$ are the vector potentials of the i^{th} particle coming from the propagation and confinement equation respectively (remember that the composite fermions and bosons of Table 3 have a mixed contribution from the n_l and $i_l + k_l$ functions). For a single particle, characterized by a total vector potential $\vec{A} = \vec{A}_p + \vec{A}_c$, the potential energy reduces to

$$U = \left(\frac{1}{2}\vec{A}.\vec{A}\right)^2 - \frac{1}{2}\vec{A}.\vec{A}. \tag{68}$$

As seen in Fig. 4, ***the nonlinear potential provides for each isolated particle a kind of Mexican hat profile*** [1]. Considering instead a high particle density so that no particle can be considered as isolated, the potential for the same particle may be written as

$$U = \left[\frac{1}{2}\vec{A}.\vec{A} + \sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]^2 - \frac{1}{2}\vec{A}.\vec{A} - \sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 \tag{69}$$

removing the Mexican hat profile as shown in Fig. 4. This clearly reminds the physical content and motivation of the Higgs mechanism [1], although, here, ***any particle will contribute to the spontaneous symmetry breaking [1, 6] and not only a given boson field (the δ-Higgs boson in DuDy theory)***.

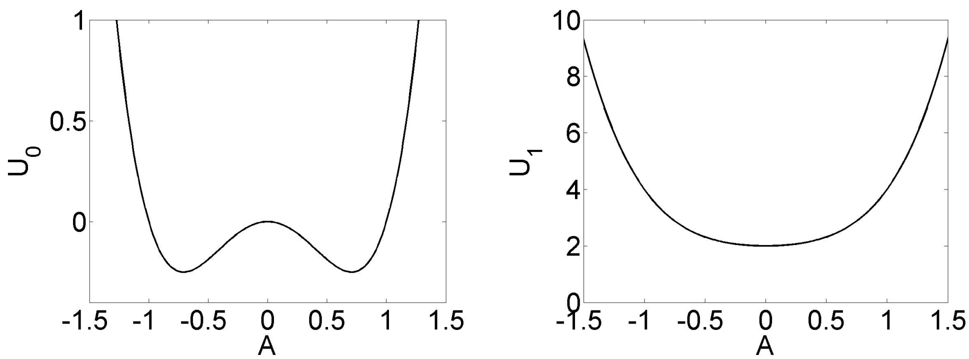

Figure 4: Schematic representation of the potential energy with $U_N = \left(\vec{A}.\vec{A}+N\right)^2 - \vec{A}.\vec{A} - N$ *for* $N=0$ *and* $N=1$.

For the following discussion, the Lagrangian approach will be favored as in the field theories used in the Standard Model [6]. The propagation and confinement Lagrangian densities write:

$$\begin{aligned}
\mathcal{L}_p &= \sum_i \left[\frac{1}{2}\left(\frac{\partial \vec{A}_p^i}{\omega_i \partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_p^i}{k_i \partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_p^i}{k_i \partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial \vec{A}_p^i}{k_i \partial z}\right)^2\right] \qquad (70)\\
&- \left[\sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]^2 + \sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - \sum_i \vec{j}_p^i.\vec{A}_p^i\\
\mathcal{L}_c &= \sum_i \left[\frac{1}{2}\left(\frac{\partial \vec{A}_c^i}{\omega_i \partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_c^i}{k_i \partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_c^i}{k_i \partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial \vec{A}_c^i}{k_i \partial z}\right)^2\right] \qquad (71)\\
&- \left[\sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]^2 + \sum_i \frac{1}{2}\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - \sum_i \vec{j}_c^i.\vec{A}_c^i
\end{aligned}$$

New propagation and confinement equations can be derived from the previous Lagrangians using the standard procedure [6]:

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x^\mu)}\right] - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad (72)$$

where the field ϕ is replaced by the vector potential $\vec{A}_p^i$ of the i^{th} particle and ∂x^μ stands for the time and spatial derivatives:

$$\left[\frac{\partial^2}{\omega_i^2\partial t^2} - \frac{1}{k_i^2}\Delta\right]\vec{A}_p^i + \left[\sum_i\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - 1\right]\vec{A}_p^i - \vec{A}_c^i = \vec{j}_p^i \tag{73}$$

$$\left[\frac{\partial^2}{\omega_i^2\partial t^2} + \frac{1}{k_i^2}\Delta\right]\vec{A}_c^i + \left[\sum_i\left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - 1\right]\vec{A}_c^i - \vec{A}_p^i = \vec{j}_c^i \tag{74}$$

The very fundamental point here is that ***the propagation and confinement equations are now nonlinear and coupled***. This will have tremendous consequences as shown in the following sections.

4.2 Particle pairing and generations of particles

The admixture of propagation and confinement properties for a single particle state has already been employed (although indirectly) in the definition of composite fermions and bosons (see section 3.2): the vector potentials were added in order to fulfill the $1/r$ (boson) and $1/r^2$ (fermion) criteria. This has finally lead to the elementary particles of the DuDy theory listed in Table 3. It is therefore not surprising to see interaction terms involving sums of propagation and confinement vector potentials or to end up with coupled propagation and confinement equations (see Eqs. 73 and 74). The non-linearity is however a completely new feature in the present theory, necessitating a close inspection in order to analyze its implications. In particular, the quadratic term is in principle capable of generating harmonic responses in time or space, which indeed is an unwanted feature: it would simply annihilate the entire identification of elementary particles and fundamental forces discussed in section 3. Hence, if new properties are definitely desired, they are expected to preserve the main findings presented up to now.

4.2.1 Spin pairing

The first and easiest non-linear effect to investigate is related to the time evolution and therefore to the notion of spin in the present theory (see section 3.5). The key point here is that the vector potentials $\vec{A}_{p/c}^i$ appear in a squared form, and not as a squared modulus, in the nonlinear wave-equations (Eqs. 73 and 74). Hence, considering an $\exp \pm i\omega t$ time dependence, the nonlinear terms will be proportional to $\exp \pm 2i\omega t$ and therefore oscillating in time at the harmonic frequency 2ω. This is incompatible with the identification of spins with the $\exp \pm i\omega t$ solutions as proposed in section 3.5.

An alternative approach is to use the linear combination of $\exp \pm i\omega t$***:*** $\cos\omega t$ ***and*** $\sin\omega t$ ***as a basis for the new spin states***. If the squared functions still oscillate in time, their sum $(\cos\omega t)^2 + (\sin\omega t)^2$ does not. Hence, pairing the two new spin states

$$\mathbf{A}_{\uparrow\downarrow} = \left|\begin{matrix}\vec{A}\,[\cos\omega t]\\ \vec{A}\,[\sin\omega t]\end{matrix}\right. \tag{75}$$

leads to a time invariant nonlinear term in the propagation and confinement wave-equations (Eqs. 73 and 74), thanks to the sum over the particles of the squared vector potentials. As a consequence, the notion of spin is preserved. I would like to emphasize that by "pairing" I mean gathering two particles having distinct quantum numbers (here the spin). It fundamentally differs from creating a new particle state by summing their vector potentials: here, the two particles do cohabit, explaining the notation with a bold $\mathbf{A}_{\uparrow\downarrow} = \left| \begin{array}{l} \vec{A}_1 \\ \vec{A}_2 \end{array} \right.$ instead of creating a new vector potential $\vec{A} = \vec{A}_1 + \vec{A}_2$. This has utmost implications largely over-heading the context of the present discussion: for example, ***stable δ-electron configurations appear with paired spins*** as it is the case in nature for isolated atoms or for molecules, where the bounded atoms share electron pairs.

4.2.2 Angular momentum and colored charge pairing

The nonlinear terms in the propagation and confinement wave-equations (Eqs. 73 and 74) also impact the use of spherical harmonics $Y_l^m(\theta,\varphi)$ and by extension the notion of angular momenta: the azimuthal and total angular momenta might no longer be good quantum numbers for an isolated particle. This would sound as a crippling weakness of the present theory. Fortunately, there is a complete analogy between the $\exp \pm im\varphi$ factor of the spherical harmonics and the time harmonic solutions $\exp \pm i\omega t$ discussed in the previous section. Hence, the same procedure can be employed. The linear combinations of the eigen-functions lead here to the real (even and odd) spherical harmonics

$$Y_l^{m,e}(\theta,\varphi) = \frac{Y_l^m(\theta,\varphi) + Y_l^{-m}(\theta,\varphi)}{2} \tag{76}$$

$$Y_l^{m,o}(\theta,\varphi) = \frac{Y_l^m(\theta,\varphi) - Y_l^{-m}(\theta,\varphi)}{2i}. \tag{77}$$

The definitions of the vector potentials (Eqs. 40-42) and the corresponding colored charges and magnetons (see section 3.3) have to be revisited accordingly, but there is no conceptual change since this is just a rotation in the function basis. Owing to the partial normalization used for spherical harmonics (Eq. 43), even and odd spherical harmonics fulfill the relation

$$\left[Y_l^0(\theta,\phi)\right]^2 + \sum_{1\le m\le l} \left[Y_l^{m,e}(\theta,\phi)\right]^2 + \sum_{1\le m\le l} \left[Y_l^{m,o}(\theta,\phi)\right]^2 = 1. \tag{78}$$

As a consequence, the nonlinear terms in the propagation and confinement wave-equations (Eqs. 73 and 74) will have no angular dependence provided that ***the elementary fermions with $l \geq 1$, as the δ-quarks of the DuDy theory, do not exist isolated but are paired in composite particles having all allowed colors for a***

given l. For the δ-quarks with $l=1$, this writes

$$\mathbf{A}_{\delta-proton} = \left| \begin{array}{l} \frac{1}{8\pi k}\vec{\nabla}\left[\left(-n_1+(i_1+k_1)\right)Y_1^0\ \ (\theta,\phi)\right] \\ \frac{1}{8\pi k}\vec{\nabla}\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,e}(\theta,\phi)\right] \\ \frac{1}{8\pi k}\vec{\nabla}\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,o}(\theta,\phi)\right] \end{array}\right. \tag{79}$$

$$\mathbf{A}_{\delta-neutron} = \left| \begin{array}{l} \frac{1}{8\pi k}\vec{\nabla}\times\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^0\ \ (\theta,\phi)\vec{r}\right] \\ \frac{1}{8\pi k}\vec{\nabla}\times\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,e}(\theta,\phi)\vec{r}\right] \\ \frac{1}{8\pi k}\vec{\nabla}\times\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,o}(\theta,\phi)\vec{r}\right] \end{array}\right. \tag{80}$$

$$\mathbf{A}_{\delta-neutrino} = \left| \begin{array}{l} \frac{1}{8\pi}\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^0\ \ (\theta,\phi)\vec{r}\right] \\ \frac{1}{8\pi}\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,e}(\theta,\phi)\vec{r}\right] \\ \frac{1}{8\pi}\vec{\nabla}\times\left[\left(-n_1+(i_1+k_1)\right)Y_1^{1,o}(\theta,\phi)\vec{r}\right] \end{array}\right. \tag{81}$$

This color pairing explains why δ-protons and δ-neutrons are composed of three δ-quarks in the present theory, while δ-electrons are not. However, there is a feature, which is not present in the Standard Model [1, 6]: δ-neutrinos are also composite particles built with the three corresponding δ-neutrino quarks.

The necessity of pairing all even and odd spherical harmonics for a given l finds astonishing resonances in chemistry: atoms are known to share electron pairs with other atoms until they reach a complete electronic shell, corresponding to fully occupied orbitals (all m values for a given l). In contrast, rare gas atoms having already fully occupied orbitals (all electrons are paired) do not need forming bindings as they are stable by their own.

4.2.3 Generation of particles

Elementary fermion pairing has been envisioned in the previous sections for the spin (time dependence) and dual colored charges (angular dependance) separately. Taken together, they support the standard interpretation of the unique He^4 stability, since the latter are composed of

- two electrons (spin pairing),
- two protons (spin pairing) composed of three δ-proton quarks (color pairing),
- two neutrons (spin pairing) composed of three δ-neutron quarks (color pairing).

(Note however that the quark content is not the same as in the Standard Model [1, 6]). Yet, none of these pairing schemes explains the existence of particle generations [1].

In order to simplify the investigation of the processes at the origin of particle generations, I will focus on the δ-electrons having $l=0$ and consider that the two electron spins have been paired (there is no time dependance in the nonlinear term as shown in section 4.2.1). Hence, the propagation and confinement wave-equations (Eqs. 73 and 74) reduce to

$$\left[-1-\frac{1}{k^2}\Delta\right]\vec{A}_p+\left[\left(\vec{A}_p+\vec{A}_c\right)^2-1\right]\vec{A}_p-\vec{A}_c=\vec{0} \tag{82}$$

$$\left[+1-\frac{1}{k^2}\Delta\right]\vec{A}_c-\left[\left(\vec{A}_p+\vec{A}_c\right)^2+1\right]\vec{A}_c+\vec{A}_p=\vec{0} \tag{83}$$

where $\frac{\partial^2}{\omega^2\partial t^2}$ has been replaced by -1 and the signs of the confinement equation have been reversed and the source terms removed for simplicity. Noting that the vector potentials are purely radial for $l=0$ and that $\Delta\vec{A}=\Delta\left(A(r)\vec{e}_r\right)=\left(\Delta A(r)-2A(r)/r^2\right)\vec{e}_r$, the previous equations can be further simplified to

$$\left[-2-\frac{1}{k^2}\Delta+\frac{2}{k^2r^2}+\left(A_p+A_c\right)^2\right]A_p-A_c=0 \tag{84}$$

$$\left[+2-\frac{1}{k^2}\Delta+\frac{2}{k^2r^2}-\left(A_p+A_c\right)^2\right]A_c+A_p=0 \tag{85}$$

where the r-dependence of the vector potentials $\vec{A}$ has been removed for more clarity. Despite their apparent simplicity, I did not manage to solve analytically the previous coupled equations. Nevertheless, many conclusions can be drawn using simple arguments. First, the nonlinear terms being proportional to the squared vector potentials, they are negligible at large distances for both composite bosons and fermions. Hence, the r dependence of vector potentials (arising from the n_l and i_l+k_l) is not altered down to some length scales short compared to the wavelength. More precisely, equaling the two "potential" terms

$$\frac{2}{k^2r^2}=\left(\frac{1}{4\pi k^2r^2}\right)^2 \quad\Leftrightarrow\quad r_\delta=\frac{\lambda}{2\sqrt{2}(2\pi)^2}<\frac{\lambda}{100} \tag{86}$$

which is far beyond the near-field regime. Hence, ***all the elementary particles and fundamental forces identified in section 3 are preserved. However, below this critical length scale r_δ, completely new physics shows up***.

In order to anticipate the physical impact of the nonlinear terms, Eqs. 84 and 85 can be compared to the "standard" Hamiltonian expression:

$$\left[-\hbar\omega-\frac{\hbar^2}{2m}\Delta+V(r)\right]\varphi=0. \tag{87}$$

By analogy, ***the nonlinear terms will behave either as a repulsive potential (propagation equation) or as an attractive potential (confinement equation)***. Hence,

- the electric field distribution associated with the propagation equation will be repulsed from the origin $r = 0$ as sketched in Fig. 5. Basically, this will break the $1/r^2$ dependence characterizing a point like charge, meaning that ***the nonlinear potential will give an effective size to the electric charge***, as observed for electrons and protons [1].

- the nonlinear term in the confinement equation playing the role of an attractive potential, the corresponding electric field distribution will be "self-trapped" in the potential well. Trapping in a potential well meaning quantization, this is the way the particle generations are foreseen to appear in the DuDy theory: ***the fundamental states corresponds to the first generation of particles, and excited states are associated with the second and third generations in the present theory***. A close inspection of the nonlinear potential in the confinement equation (Eq. 85) shows that it has three roots (three A_c amplitudes canceling the potential) for sufficiently small r. This suggests that the number of generations might be limited to three, as it is the case in the Standard Model [1, 6].

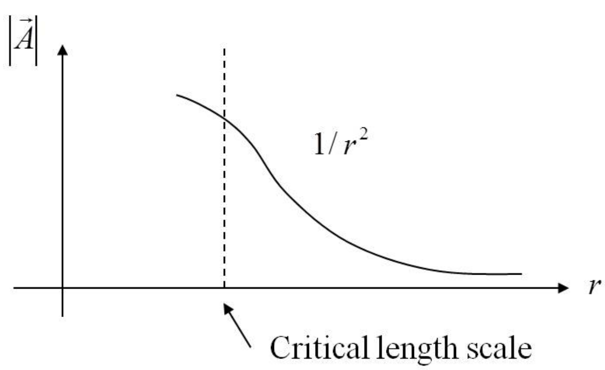

Figure 5: Effect of the nonlinear potential on the vector potential amplitude.

Interestingly, the values of the charges and masses, as introduced in section 3.3, are not affected in the present theory by the existence of several generations: they have been merged in a single entity (the colored charges) associated with the $1/r^2$ dependence at larger scale, which is preserved by the nonlinear terms as discussed previously. In other words, ***all particle generations have the same dual colored charge in the DuDy theory. But the effective masses arising from the indirect interaction with the gravitons do depend on the exact spatial distribution of the vector potential (see section 3.6.1). The effective masses are therefore expected to vary from generation to generation***.

If the self trapping/quantization effect provides increasing frequencies to the composite particle generations, it also generates an unstable character to the excited states, which can desexcite by emitting pure bosons or pairs of composite boson/anti-boson of Table 3. In contrast, the ***fundamental states are infinitely stable in the present theory at least for $l = 0$ (δ-electrons) and $l = 1$ (δ-neutrinos, neutrons and protons), since they have no way to decay and fulfill the charge/magneton and energy conservation laws***. Things might be different for the δ-technicolor quarks ($l = 2$) since they can split for example in δ-quarks with ($l \leq 1$). This spontaneous annihilation and the fact that they might have higher energies (as suggested by the fact that the rest energy of electrons with $l = 0$ is nearly 1823 times smaller that the rest energy of protons, having $l = 1$) could explain why these composite particles have not been observed yet.

All composite fermions and bosons of Table 3 having distinct r dependence (they involve $\vec{\nabla}$, $\vec{\nabla}\times$ and $\vec{\nabla}\times\vec{\nabla}\times$ operators acting on n_l and $i_l + k_l$), they do not experience in the same way the self trapping/quantization effect. This has a fundamental consequence as, ***without any adjustable coupling constant λ, it is possible to account for a large variety of masses, as it is expected for the known fundamental particles of the Standard Model*** [1, 6]. This would be a great step toward unification of Physics if all known masses can be reproduced as it would dramatically reduce the number of unknowns compared to the Standard Model. Unfortunately, this is out of reach in the present analytical approach since the nonlinear coupled confinement and propagation equations (Eqs. 84 and 85) have to be solved simultaneously (it is even worse for $l > 0$ since the colored charges have to be paired). However, this represents an accessible challenge for numerical simulations and would be a definitive test for the validity of the present theory.

The last but not least consequence of the nonlinear terms in the confinement wave-equation (Eq. 74) is related to the behavior of strong forces. The back-action applied by the nonlinear terms onto the vector potentials holds for $r \to 0$ but also for $r \to \infty$. More precisely, the i_l and k_l are no more allowed to diverge as $e^{kr}/2kr$ [7]; their amplitude are regularized by the attractive nonlinear potential. This can be seen from Eq. 85 for $l = 0$ particles by setting $A_p = 0$: the exponential growth of the vector potential is contradicted by the negative feedback provided by $-A_c^2$. Hence, ***quarks and gluons are no more linked by an infinitely growing potential with their spatial separation: the r-dependence of the strong interaction is not the same at small and large distances***. This seems to be compatible with the behavior of the strong interaction of the standard model [1, 6].

4.2.4 The singular case of pure bosons

The previous sections were essentially oriented toward the nonlinear effects on composite fermions and bosons. Nothing was said on pure bosons. Pure bosons are very singular since, following the classification of Table 3, only j_l functions enter in their vector potentials definition. Hence, the nonlinear confinement equation (Eq. 74) will be disregarded in the present section, so that the nonlinear propagation equation is modified in

$$\left[\frac{\partial^2}{\omega_i^2 \partial t^2} - \frac{1}{k_i^2}\Delta\right] \vec{A}_p^i + \left[\sum_i \left(\vec{A}_p^i\right)^2 - 1\right] \vec{A}_p^i = \vec{0} \tag{88}$$

(Remember that there is no source term associated with pure bosons as discussed in section 3.4). Written in this way the pure bosons experience a nonlinear self-coupling, which sounds to be a highly unexpected feature in the context of the Standard Model. Might the pairing schemes used for composite fermions and bosons solve this issue? Is there new physics to discover?

Let us first consider the case of δ-photons. Pairing the photon spins allows suppressing the time dependence of the quadratic term (see section 4.2.1). The angular dependence of the latter can be removed by pairing the spherical harmonics as done in section 4.2.2. Finally, pairing the $\vec{\nabla}\times$ and $\vec{\nabla}\times\vec{\nabla}\times$ vector potentials (see Eqs. 41 and 42) might help in removing the spatial oscillations induced by the j_l functions, but their $1/r$ dependence at large distance remains [7]. Hence, the pairing schemes envisioned up to now leads here to an impasse; a new approach is needed. For this proposal, pure bosons will be linearly combined in new elementary particles described as plane waves as shown in Table 4.

Old pure bosons		***New pure bosons***	
δ-graviton		δ-graviton	
$\vec{\nabla} \mid j_l$	with $l = 0..\infty$	$\vec{e}_0.\exp(i\vec{k}.\vec{r})$	with $\vec{e}_0 = \frac{1}{k}\vec{k}$
δ-photons		δ-photons	
$\vec{\nabla}\times \mid j_l$	with $l = 1..\infty$	$\vec{e}_1.\exp(i\vec{k}.\vec{r})$	with $\vec{e}_1 \perp \vec{k}$
$\vec{\nabla}\times\vec{\nabla}\times \mid j_l$	with $l = 1..\infty$	$\vec{e}_2.\exp(i\vec{k}.\vec{r})$	with $\vec{e}_2 = \frac{1}{k}\vec{k}\times\vec{e}_1$

Table 4: New definition of elementary pure bosons as longitudinal and transverse plane-waves.

As for spherical harmonics in section 4.2.2, there is no fundamental concept change since the two pure boson descriptions are related by a "rotation" of the function basis. But even with these new definitions, the pure bosons do not lead to a quadratic coupling term independent of time and space. However, using left (L) and right (R) circularly polarized δ-photons

$$\vec{e_R} = \vec{e}_1.\cos(\omega t + \vec{k}.\vec{r}) + \vec{e}_2.\sin(\omega t + \vec{k}.\vec{r}) \tag{89}$$

$$\vec{e_L} = \vec{e}_1.\cos(\omega t + \vec{k}.\vec{r}) - \vec{e}_2.\sin(\omega t + \vec{k}.\vec{r}) \tag{90}$$

allows removing any oscillating components in the quadratic terms of Eq. 88. Hence, ***the transverse δ-photons are described by circularly polarized plane waves having a definite spinning effect. But more importantly, the photon wave-function amplitude is normalized by the nonlinear coupling Hamiltonian (Eq. 67), and not by any ad-hoc condition, so that the δ-photon naturally appears as a quantum***.

Consider now that N identical δ-photons occupy the same quantum state described by the plane wave $\vec{A}_p^{(N)}$. This plane wave can be expressed as $\vec{A}_p^{(N)} = A_p^{(N)}\vec{A}_p^{(1)}$ where $A_p^{(N)}$ is a normalization constant and $\vec{A}_p^{(1)}$ refers to the single photon plane wave (Eqs. 89 and 90). Inserting this solution into the propagation equation (Eq. 88), the two first terms cancel out leading to

$$\sum_n \left[\vec{A}_p^{(N)}\right]^2 - 1 = 0 \tag{91}$$

i.e. $A_p^{(N)} = 1/\sqrt{N}$ since $\vec{A}_p^{(1)}$ has a unity amplitude. Hence, the total vector potential $\vec{A}_p^N$ associated with N identical photons is given by

$$\vec{A}_p^N = \sum_n \vec{A}_p^{(N)} = \sqrt{N}\vec{A}_p^{(1)} = \sqrt{N}\vec{A}_p^1 \tag{92}$$

which is precisely the expected result in quantum electro-dynamics [11]. Hence, ***as surprising as it might be, the nonlinear term in the photon propagation equation allows accounting for many results, such as the photon anti-bunching in Hong-Ou-Mandel experiments, for example*** [12]. However, I am reluctant in introducing creation and annihilation operators for the δ-photons: the associated vector potentials being fully delocalized, this would directly lead to locality paradoxes. This strongly differ from composite particles whom dual colored sources are point-like objects (see section 3.3), allowing to define creation and annihilation operators as done in section 3.4. Hence, a δ-photon quantum will be emitted or absorbed as a whole by the composite fermions system thanks to the creation/annihilation operators of the dual colored sources. Note that this processes cannot be instantaneous since the photon energy, which is delocalized in space, has to be transferred in order to generate a composite fermions transition (such as an electronic transition). In other words, composite fermions occupation numbers do not make an instantaneous jump so that the creation and annihilation operators introduced in Eqs. 55-56 are not generally speaking Dirac delta functions of time.

Longitudinal δ-gravitons can now be investigated. They differ from the δ-photons since they have only one polarization. Hence, a real pairing scheme is necessary here, so that ***the resulting δ-graviton is a composite particle in the DuDy theory***

$$\mathbf{A}_{\delta-graviton} = \left| \begin{array}{l} \frac{1}{k}\vec{k}.\cos(\omega t + \vec{k}.\vec{r}) \\ \frac{1}{k}\vec{k}.\sin(\omega t + \vec{k}.\vec{r}) \end{array} \right. \tag{93}$$

This composite δ-graviton now leads to a constant quadratic term in the pure boson wave-equation (Eq. 88) and is subject to the same normalization rule as the δ-photons. However, the procedure followed here has "broken" the symmetry between δ-photons and δ-graviton: while the δ-photons are elementary particles described by simple vectorized wave-functions $\vec{A}_{ph}$, the δ-graviton $\mathbf{A}_g$ is a composite particle involving two elementary longitudinal vector fields. The difference between photons and gravitons is indeed not a surprise since it is also present in many theories, where they are described by mass-less spin-1 and spin-2 elementary particles respectively [1].

5 DuDy and the cosmology

Unifying physics at infinitely small and infinitely large length scales requires to describe particles physics and cosmology within the same framework. As seen in sections 3.6.1 and 4.2.4, candidates for the gravitational interaction and its corresponding boson have been obtained in the present theory from the dual propagation and confinement dynamics. The corresponding equations (Eqs. 73 and 74) have already shown unique properties, suggesting to push forward their investigation and see how they behave at large length scales. For this proposal, I will start from the smallest structures (the black-holes) and progressively zoom out with galaxies, voids, universe expansion and microwave background, which at the end will ask the question of the origin, the big-bang, ultimately linking infinitely small and infinitely large length scales.

5.1 Black-holes

Black-holes correspond to singularities predicted by the general relativity [15]. They are believed to be hosted in many galaxies including our Milky Way, and have masses ranging from 10 to 10^{10} sun masses. Despite their mass variability, black-holes share two main properties:

- a black-hole is a region of space from which nothing, including light, can escape (event horizon).
- a black-hole has only three independent physical properties: mass, charge, and angular momentum (no hair theorem).

How can this be accounted for by the DuDy theory? Do hyper-massive particles emerge from the derived quantum framework? How do they form from standard matter made of composite particles?

Let us first investigate the potential existence of hyper-massive particles and restrict the discussion to the simplest case corresponding to $l = 0$ angular momentum. For this proposal, the simplified propagation and confinement equations (Eqs. 84-85) can be used. In contrast with what has been done for the pure bosons (see section 4.2.4), we will consider here that the propagation vector potential is zero and therefore that only the confinement equation has to be taken into account:

$$\left[+2 - \frac{1}{k^2}\Delta + \frac{2}{k^2 r^2} - (A_c)^2\right] A_c = 0 \tag{94}$$

(Remember that the source term has been removed for simplicity). Written is this way, there is a clear connection between the non-linear confinement equation, the nonlinear Schrodinger equation [13] and the Gross-Pitaevskii equation [14] leading to the formation of Bose-Einstein condensate:

$$\left(-\mu - \frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + U_0|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = 0 \tag{95}$$

The corresponding solutions have already been shown to behave as black-holes [16]. Hence, in the present theory, ***δ-black-holes will be described by Bose-Einstein condensates of massive elementary particles satisfying the nonlinear confinement equation***. Even if the general relativity [2] has not been invoked to generate the black-hole state, Einstein's theories do play a fundamental role in the present work, from the very start with the $E^2 = p^2c^2 + m^2c^4$ relation which has fathered the idea of dual dynamics (see section 2.3) to the present discussion with the Bose-Einstein condensation. Is there other connections with Einstein's general relativity? The model of Bose-Einstein condensation for δ-black-hole has many interesting features:

- ***the black-hole singularity of the general relativity corresponds to the fact that the overall black-hole mass is contained in a region of space with zero volume [15]. It is replaced in the present theory by the source terms originating from the particle-source duality for composite particles*** (see section 3.4). Hence, in both cases, black-holes are generated by point-like entities.

- The solution of the propagation equation (Eq. 73) being not orthogonal to those of the confinement equation (Eq.74), ***δ-the photons $\vec{A}^{ph}$ do interact with the wave component $\vec{A}^{bh}$ of δ-black-holes. More precisely, the wave component of the δ-black-hole is capable of deviating the δ-photons, as black-holes do in general relativity. Hence, a kind of Einstein's gravitational lensing [2] can be accounted for in the present theory***. I would like to further emphasize this very fundamental point linking the present theory with the General Relativity. In general relativity [2], the space-time entity is "deformed" by the gravity induced by a point-like object (the black-hole). ***In the present theory, it is the wave which is deformed around the source term of the δ-black-hole, while the space-time entity remains completely flat***, as sketched in Fig. 6.

- the amplitude of the vector potential being proportional to the square root of the particle number $\sqrt{N}$ (cf. section 2.2), it can be huge in the δ-black-hole condensate. The probability for an incoming particle or photon to be absorbed tends therefore to 1. This provides an analogous of the event horizon of the general relativity [15]: ***the event horizon in the present theory is the sphere defined by a unitary absorption probability***.

- owing to the particle pairing enforced by the nonlinear coupling Hamiltonian (see section 4), ***stable δ-black-holes condensates should exist only with paired spins and paired colors. As a consequence, the squared amplitude of their vector potentials has no more time nor angular dependence as discussed in section 4. This provides an analogous of the "no hair theorem"*** [15]. Following Table 2, different types of vector potentials might be involved leading to different black-holes properties. In particular, black-holes with neither mass nor charge but dual colored magnetons are allowed, owing to their transverse character.

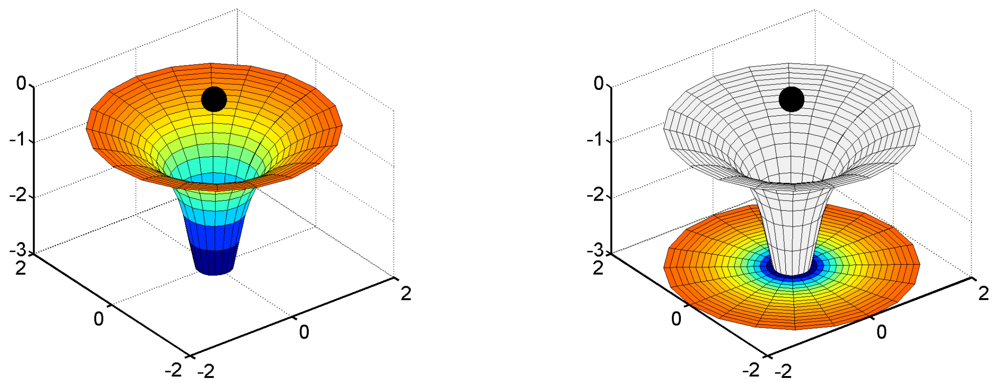

Figure 6: Schematic representation of the space (in colors) deformed by a massive particle in the framework of general relativity (left) and a perfectly flat space with a wave-function (in gray) deformed by the point-like source (right).

5.2 Quasars and blazars

From the theory of differential equations it is known that there are two independent solutions for a second order differential equation. For this reason, $j_l(kr)$ and $n_l(kr)$ functions were used for the propagation equation (Eq. 30), whereas $i_l(kr)$ and $k_l(kr)$ functions were employed for the confinement equation (Eq. 31). In order to obtain the fermions listed in Table 3, diverging and usually forbidden spherical (modified) Bessel functions where considered. Only a few of them were finally selected and mixed to satisfy the $1/r$ (composite bosons) and $1/r^2$ (composite fermions) criteria for the vector potentials (see section 3.2). The same hold for the angular dependence: ***there is a second set of independent angular functions known as the associated Legendre functions of the second kind*** [7]. Hence the following substitution has to be considered:

$$P_{lm}(\cos\theta) \quad \leftrightarrow \quad Q_{lm}(\cos\theta) \tag{96}$$

where P_{lm} are the associated Legendre polynomials and Q_{lm} the associated Legendre functions of the second kind. As shown in Fig. 7, the Q_{lm} solutions diverge at $\theta = [0, \pi]$. This explains why they are usually rejected. However, as seen previously for the spherical Bessel functions, a careful look at usually forbidden function might greatly help understanding specific physical properties. Applying the same reasoning as in section 3.2

and taking into account that $dV = r^2 dr \sin\theta d\theta d\varphi$ the amplitude of the vector potential should not vary faster than $1/\sin\theta$. After some algebra, one can show that the only allowed solutions have $m = 0$ (the value of m is therefore omitted):

$$Q_0(\cos\theta), \quad Q_1(\cos\theta), \quad Q_2(\cos\theta) \tag{97}$$

Only the $l \leq 2$ solutions are retained here since, at least at low energy, the definition of fermions and bosons have restricted the angular momentum (see section 3.2).

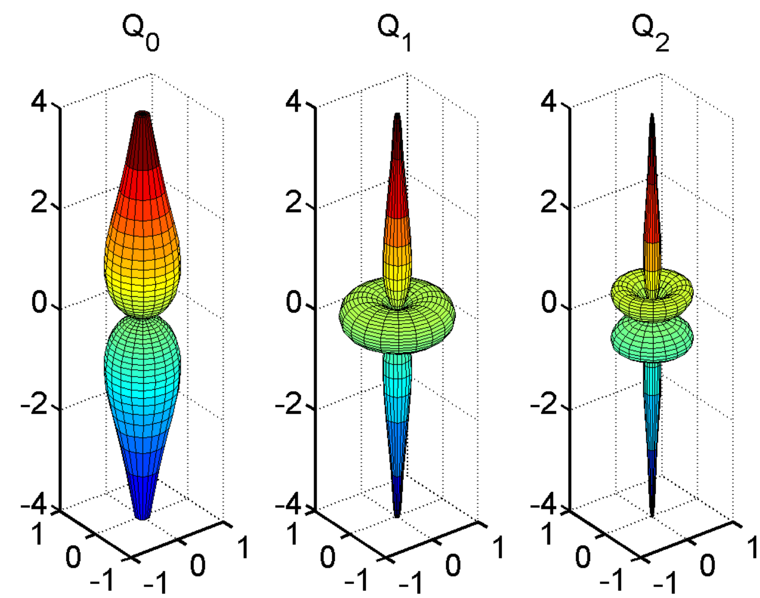

Figure 7: Legendre functions of the second kind Q_0, Q_1 and Q_2. The surface plot is given by $Q_l(\cos\theta)\vec{e}_r$ and have been truncated for $Q_l > 4$ since the Legendre functions of the second kind diverge at $\theta = 0$ and $\theta = \pi$.

As surprising as they might be, these solutions have already been observed in universe. The diverging amplitude at $\theta = [0, \pi]$ leads to the formation of jets present in many active galactic nuclei such as quasars, blazars and radio galaxies [17]. These AGN contain black-holes, i.e. a macroscopic quantum object as discussed in section 5.1. Depending on the value of l, the morphology of the "jet-particle" and its corresponding black-hole is different. Considering the $\vec{\nabla}.$ solution (Eq. 40), which is dominated by its radial component at large distance, the vector potential amplitude is directly proportional to Q_l (see Fig 7). Hence,

- for $l = 0$, the vector potential goes to zero at $\theta = \pi/2$ and exhibit two large lobes ended by two jets at $\theta = [0, \pi]$. This pattern has clear similarities with the FRII radio galaxy 3C98 [17].

- for $l = 1$, the vector potential is non zero at $\theta = \pi/2$, cancels out symmetrically and then diverges at $\theta = [0, \pi]$. This particle is therefore compatible with an accretion disc perpendicular to the jets, as observed in quasars, and blazars [17].

- for $l = 2$, the vector potential vanishes twice leading to a double accretion disc looking as inverted cups (diabolo). This unusual morphology has been observed in the HH-30 Herbig-Haro object [18]. ***This might be viewed as a proof of existence for the δ-technicolor quarks and related composite bosons, introduced in Table 3 as having $l = 2$ angular momenta.***

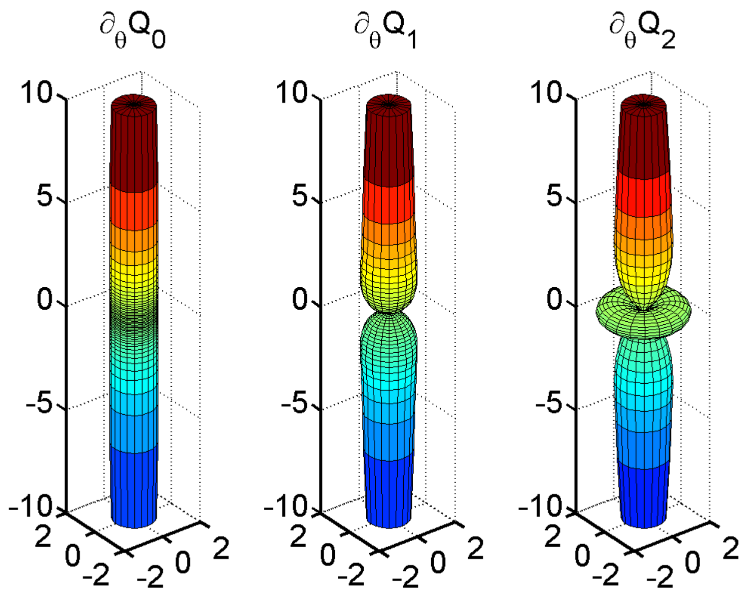

Figure 8: Derivative of the Legendre functions of the second kind Q_0, Q_1 and Q_2. The surface plot is given by $\partial_\theta Q_l\,(\cos\theta)\,\vec{e}_r$ and have been truncated for $\partial_\theta Q_l\,(\cos\theta) > 10$ since they diverge at $\theta = 0$ and $\theta = \pi$.

If we now focus on the $\vec{\nabla}\times$ and $\vec{\nabla} \times \vec{\nabla}\times$ solutions (Eqs. 41-42), their far-field component evolves as $\partial_\theta Q_l$ (see Fig. 8). In these cases,

- for $l = 1$, the vector potential generates two elongated lobs holding each a jet. It resembles the $\vec{\nabla}.$ solution with $l = 0$ but with lobes having nearly constant diameters, as observed in the radio galaxy 3C31 composed of two "plumes" [17].

- for $l = 2$, the vector potential presents an accretion disk with two large lobes compared to the $\vec{\nabla}.$ solution with $l = 1$. Each lobe still contains a jet, mimicking the morphology of the radio galaxy Centaurus A [17].

As discussed above, only $m = 0$ azimuthal angular momenta are allowed for the jet-particles. The color pairing introduced in section 4.2.2 is therefore not possible here. In contrast, the spin pairing do apply, leading to time invariant quadratic terms in the confinement equation. However, ***when additional matter falls into the δ-black-hole, the spin pairing is temporary broken. The vector potential $\vec{A}^{bh}$ describing the Bose-Einstein condensate oscillate in time until it ejects the exceeding matter in order to reach the pairing balance again. In addition, as a consequence of the quadratic terms in the confinement equation (Eq. 74), harmonics will be generated in this transient phenomenon***, as discussed in section 4.2.1. This surprising effect has indeed been observed is quasars [19].

Finally, I would like to discuss here a fundamental property of "standard" and "jet" particle condensates forming δ-black-holes. The propagation and confinement equations (Eqs. 73 and 74) for a particle at the vicinity of the Bose-Einstein Condensate $\vec{A}^{bh}$ write

$$\left[\frac{\partial^2}{\omega^2 \partial t^2} - \frac{1}{k^2}\Delta\right]\vec{A}_p + \left[\vec{A}.\vec{A} + \vec{A}^{bh}.\vec{A}^{bh} - 1\right]\vec{A}_p - \vec{A}_c = \vec{j}_p \tag{98}$$

$$\left[\frac{\partial^2}{\omega^2 \partial t^2} + \frac{1}{k^2}\Delta\right]\vec{A}_c + \left[\vec{A}.\vec{A} + \vec{A}^{bh}.\vec{A}^{bh} - 1\right]\vec{A}_c - \vec{A}_p = \vec{j}_c \tag{99}$$

The amplitude of the vector potential $\vec{A}^{bh}$ being huge as discussed in section 5.1, the retardation effects (temporal derivative) and other coupling terms can be completely neglected, leading to the simplified equations:

$$-\frac{1}{k^2}\Delta\vec{A}_p + \left(\vec{A}^{bh}\right)^2\vec{A}_p = \vec{j}_p \tag{100}$$

$$+\frac{1}{k^2}\Delta\vec{A}_c + \left(\vec{A}^{bh}\right)^2\vec{A}_c = \vec{j}_c \tag{101}$$

Hence, ***at the vicinity of δ-black-holes (including the accretion disks and the jets) the particle speed is not limited to the speed of light: superluminal speed is allowed in the present theory provided that the nonlinear terms dominate the particle dynamics***. If such superluminal movements are highly unexpected features in the context of the special or general relativity [2, 5], they have indeed been observed in the jets of the M87 galaxy and 3C 273 quasar [20]. In the latter case, velocities nearly one order of magnitude larger than the speed of light were measured.

5.3 Dark matter

Dark matter and dark energy are essential features in modern cosmology as they are needed to explain respectively [1]

- the rotation speeds of galaxies, the orbital velocities of galaxies in clusters and the gravitational lensing of background objects by galaxy clusters.
- the expansion of universe and the formation of large structures.

Visible matter, dark matter and dark energy are considered to contribute to ≈5%, 27% and 68% of the overall mass in universe [4]. Therefore, it is a great challenge for modern theories to explain this huge "missing mass". One way to explain the unsolved origin of dark matter and dark energy is traditionally to introduce new particles, since visible matter and gravitational lensing effects can have different locations, as shown in the MACS J0025.4-1222 cluster [21]. For this proposal, additional allowed vector potentials

Expression	Mass	Charge	Mass magnet.	Charge magnet.	Type of particle
$\vec{\nabla} \mid n_0$	no	yes	no	no	non-baryonic
$\vec{\nabla} \mid (i_0 + k_0)$	yes	no	no	no	non-baryonic
$\frac{1}{2}\vec{\nabla} \mid n_0 + (i_0 + k_0)$	yes	yes	no	no	non-baryonic
$\vec{\nabla} \times \mid n_1$	no	no	no	yes	non-baryonic
$\vec{\nabla} \times \mid (i_1 + k_1)$	no	no	yes	no	non-baryonic
$\frac{1}{2}\vec{\nabla} \times \mid n_1 + (i_1 + k_1)$	no	no	yes	yes	non-baryonic
$\frac{1}{2}\vec{\nabla} \mid -n_1 + (i_1 + k_1)$	yes	yes	no	no	baryonic
$\frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_1 + (i_1 + k_1)$	no	no	yes	yes	baryonic
$\frac{1}{2}\vec{\nabla} \mid (i_1 + k_1) + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid n_1$	yes	no	no	yes	non-baryonic
$\frac{1}{2}\vec{\nabla} \mid n_1 + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid (i_1 + k_1)$	no	yes	yes	no	non-baryonic
$2\vec{\nabla} + \vec{\nabla} \times \vec{\nabla} \times \mid n_2$	no	yes	no	yes	non-baryonic
$2\vec{\nabla} + \vec{\nabla} \times \vec{\nabla} \times \mid (i_2 + k_2)$	yes	no	yes	no	non-baryonic
$\vec{\nabla} + \frac{1}{2}\vec{\nabla} \times \vec{\nabla} \times \mid -n_2 + (i_2 + k_2)$	yes	yes	yes	yes	non-baryonic

Table 5: Contribution of fermions to baryonic and non-baryonic matter in DuDy theory.

combinations have been added in Table 5, using the $1/r^2$ criterion for fermions (see section 3.2) and assigning their potential contribution to baryonic or non-baryonic matter. Hence, in the present theory, there is two baryonic particles for eleven non-baryonic particles types. The corresponding ratio between baryonic and non-baryonic particles is therefore $2/11 \approx 18.2\%$, to be compared to the $4.9/26.8 \approx 18.3\%$ obtained from the estimated mass ratio in universe [4]. This surprisingly good agreement is somehow fortuitous since the particle masses are not known in the present theory (see section 4.2.3). Nevertheless, the existence of "exotic" types of fermions offers an interesting framework to investigate dark matter.

The question is then how and why this dark-matter is distributed in halos centered on galaxies [1]. The main idea is that, while the mass component (originating from the i_l+k_l functions) has collapsed into the central black-hole, the corresponding charge component (corresponding to the n_l functions) has been released. Its subsequent evolution can be understood by considering the propagation equation written for the $l = 0$ modes:

$$\left[-2 - \frac{1}{k^2}\Delta + \frac{2}{k^2 r^2} + (A_p)^2\right] A_p = 0 \tag{102}$$

The sign change of the quadratic term with respect to Eq. 94, leads to the equivalent of a repulsive force. By analogy with the nonlinear Schrodinger equation [13], where the positive quadratic generates a dark soliton, ***the positive quadratic term in the propagation equation leads to Bose-Einstein condensates having a deep in the central region. This scenario is hypothesized to generate the formation of a dark-matter halo around the galaxy centers***. More precisely, black-hole/dark-matter halo are dual aspects of the same physical process originating from the dual dynamics. Appropriate simulations are required in order to support or invalidate this interpretation and especially its ability to create "dark" Bose-Einstein condensates at the galaxy scale.

5.4 Dark energy

The concept of dark energy has been invented in order to account for the accelerated expansion of universe [1]. Since then, many models have been proposed and the two leading interpretations are:

- a uniform energy density filling space homogeneously and corresponding to a cosmological constant. It is estimated by cosmologists to be on the order of 10^{-9} J/m^3 [22],
- a scalar field whose energy density can vary in time and space.

Concerning the cosmological constant, a major issue arises from the fact that most ***quantum field theories predict a huge contribution from the energy of the***

quantum vacuum, which exceeds the estimation by more than 100 orders of magnitude [22]. This is not the case in the present theory, since the vacuum energy is strictly zero due to the specific definition of nonlinear creation and annihilation operators (see section 2.2 and 3.4). Considering the cosmic microwave background [1] as a source for this cosmological constant does not help since its 10^{-14} J/m^3 density is 5 orders of magnitude too small. Hence, the cosmological constant hypothesis should be disregarded in the present theory.

The scalar field hypothesis is ruled out from the initial postulate that all particles are described by vector potentials in the present theory. Yet, a tentative approach would be to consider the $\vec{\nabla}.$ solution with the $j_0(kr)$ function (see Eq. 40). Provided that the wavelength $\lambda = 2\pi/k$ is by far larger than the known universe size, it would be associated with a uniform divergence, potentially explaining the universe expansion: at any point in the near universe, the vector potential can be locally described by a constant value plus an additional diverging vector field, pushing all particles away. Including the time dependence would even explain the actual accelerated expansion. At some point, the vector potential amplitude would reach its maximum, decrease and finally change its sign, leading to a contraction of universe concluded by a big crunch (and eventually to a new big-bang, etc.). This sounds promising, but it would assume a pre-existing vector potential extending over the entire universe (in the present theory the big-bang might only have "impulsively" excited a given mode which is propagating away as a wave does at a water surface hit by a rock, see section 3.4). This is unsatisfactory, since the origin of the pre-existing potential would remain mysterious.

There is another possibility: anti-gravity. It is generally disregarded since negative inertial masses lead to a "runaway motion" [9] which appear unphysical. In addition, quoting the same reference: "*Bubble chamber experiments are often cited as evidence that anti-particles have a positive inertial mass equivalent to their normal counterparts, but a reversed electric charge. In these experiments, the chamber is subjected to a constant magnetic field which causes charged particles to travel in helical paths. The radius and direction of these paths correspond to the ratio of electric charge to inertial mass. Particle/anti-particle pairs are observed to travel in helices with opposite directions, but identical radii. Certainly, this observation implies that their ratios differ only in sign, but it does not make clear whether it is charge or inertial mass which is negative. However, particle/anti-particle pairs are observed to electrically attract one another. This behavior implies that both have positive inertial mass and opposite charges; if the reverse were true, then the particle with positive inertial mass would be repelled from its anti-particle partner.*"

All arguments presented in the previous paragraph indirectly rely on the equivalence principle between gravitational and inertial masses [10]. It has been partially analyzed in section 3.6.1. We came at the conclusion that the long range gravitational interaction is due to the indirect coupling between masses involving the pure bosons, while the point source interacts with its own wave through direct interaction, explaining the notion of inertia. But a fundamental aspect was omitted: the masses of particles and anti-particles as defined in section 3.5 have opposite signs, leading to repulsive gravitational forces (anti-gravity). In contrast, the inertial mass, traducing the fact that point particles have to push/pull their own clouds, do not depend on the mass sign (see section 3.6.1). In other words, the inertial mass is always positive while the gravitational mass can be positive or negative, despite the fact that they have the same origin. ***The equivalence principle [10] is therefore redefined in the present theory as***:

$$0 < m_{inertial} = \pm m_{gravitational}. \tag{103}$$

Hence, particles and anti-particles have the same inertial mass but opposite gravitational masses, removing all issues with negative masses [9].

The consequences of this new equivalence principle are of utmost importance: ***anti-gravity is responsible in the present theory for the separation of matter and anti-matter in different region of universe***. This is compatible with the observed asymmetry between baryons and anti-baryons [1]: since two anti-particles do attract each other, have the same inertial mass and interact with light as particles do, there is no way to distinguish galaxies from anti-galaxies in the present theory. However, invoking anti-gravity between particles and anti-particles, large structures may form spontaneously in universe [23] in order to minimize the repulsive force, leading to the apparition of voids, i.e. regions of universe with rarefied matter surrounded by walls of matter. This will be discussed in detail in section 5.5.

Consider now a collection of "packed" voids. Each void tends to expand due to the repulsive anti-gravity between matter and anti-matter. With the picture of an expanding foam in mind, it is clear that the distance between voids increases in time. But more importantly, an observer in a given void would see an expansion in all directions as if his own void was the center of the foam (note that the expansion rate should be small enough to prevent the observer to feel an acceleration, otherwise he would be able to deduce that he is not at the center of the foam). This picture is indeed consistent with the Hubble law written in the following way [1]:

$$\vec{v} = H_0 \vec{r} \tag{104}$$

It generates a uniform divergence $3H_0$ of the velocity $\vec{v}$ which is equivalent to a uniform local dilatation (relative volume increase $\delta V/V$) of the voids. Hence, ***in the present theory, anti-gravity will generate a negative pressure at the origin of the universe expansion, so that neither new particles nor metric expansion will be incorporated to account for dark-energy***. A quantitative evaluation of the anti-gravity effect on the universe expansion rate is therefore required to validate the present interpretation of dark-energy, but additional ingredients have to be included first, as seen in the following section.

5.5 Big-Bang

To conclude this work and summarize the key aspects of the DuDy theory, I would like to address the primordial event in universe known as the Big-Bang [1]. It is a requirement in the lambda-CMD model [1] in order to account for the existence and structure of the cosmic microwave background [1] and the abundance of hydrogen (including deuterium), helium, and lithium in universe [1]. An inflation phenomenon, corresponding to an exponential expansion of the universe during an extremely short period (less that 10^{-30} s), is further needed to explain how the universe heterogeneity arises from primordial fluctuations [1]. Does the DuDy theory generate such kinds of scenario or is it necessary to introduce new physics?

At this step, it is important to remind that the present theory uses a perfectly flat space-time based on Euclidean geometry and Galilean transformation. Hence, there is, by construction, no flatness problem [1] corresponding to the fact that the observed universe is nearly perfectly flat, i.e. with zero curvature. There is neither metric expansion as discussed at the end of the previous section, so that the space is not expanding itself as in standard inflation theories [1]. Finally, due to the specific definition of (nonlinear) creation and annihilation operators (section 2.2 and 3.4), there is no vacuum energy fluctuation as in standard field theories [6], since the vacuum energy is strictly zero, and therefore no primordial fluctuation. All this strongly constrains the DuDy theory for providing a tentative analogue of Big-Bang and inflation models.

In the present work, it is postulated that the Big-Bang arose at a given time t_0 and at a given position $\vec{r}_0$ in a pre-existing flat universe. The first consequence is that the universe is not considered as uniform as in the cosmological principle [10]. What is uniform are the physics laws and the probability that a given event might occur anywhere at anytime. In case of extremely rare events, such as the Big-Bang, this statements clearly violates the time and space invariance by accepting that something can spontaneously occur locally in time and space. But "rare" is relative to a given scale and if one considers the universe as infinite (in space and time), Big-Bangs might have been produced everywhere but at different epochs. The second consequence is that matter expansion in universe is seen in the present theory as a real expansion and not as a metric expansion. Using the picture of an expanding foam, as in the previous section, our local universe can appear as uniform, provided that the boundary of the foam is too far from us to be accessible by optical means any other wave or particle detection. Yet, a clear picture of the Big-Bang is missing.

Translational invariance has early been abandoned in the present theory in favor of isotropy, as evidence by the choice of spherical coordinates for the equations and vector potentials derivations, rather than Cartesian coordinates. Following this idea, the only requirement for the Big-Bang is to respect space and time isotropy. For this reason, all quantum numbers have to be summed up in order to fulfill the dual charge, momenta and spin conservation. In addition, a full symmetry between particles and anti-particles is necessary so that the Big-Bang is described by the following equation

$$\sum_{\vec{\forall},l,m,\uparrow\downarrow,\oplus\ominus} \vec{j} = \vec{0} \tag{105}$$

where $\vec{j}$ are the source terms of section 3.4. $\vec{\forall}$ means that all vector potentials are involved (see section 3.1), $\uparrow\downarrow$ stems for the spin states (see section 4.2.1) and $\oplus\ominus$ for the particles and anti-particles respectively (remember the π phase shift introduced in section 3.5). The summation implies an infinite number of particles created at the same time and at the same position with all possible angular momenta (l, m). This event is of course very rare. In addition, since particles and anti-particles are created at the same place, they may annihilate instantaneously. This is an unexpected feature, suggesting that something is missing.

The present theory being based on coupled dual equations, coupled dual initial conditions are required for the Big-Bang. As a consequence, the previous equation needs to be rewritten for propagation p and confinement c source terms:

$$\sum_{\vec{\forall},l,m,\uparrow\downarrow,\oplus\ominus} \vec{j}_p\left(\vec{r}_p\right) = \vec{0} \tag{106}$$

$$\sum_{\vec{\forall},l,m,\uparrow\downarrow,\oplus\ominus} \vec{j}_c\left(\vec{r}_c\right) = \vec{0} \tag{107}$$

where two separate creation positions $\vec{r}_p$ and $\vec{r}_c$ have been introduced. Then, ***in order to avoid an instantaneous annihilation of the produced particles, the symmetry has to be locally broken, i.e.***

$$\vec{r}_p - \vec{r}_c \approx \vec{0} \tag{108}$$

If the propagation and confinement source terms are close enough, they may interact before annihilating themselves. More precisely, immediately after their creation, the source terms start emitting their associated wave (cf. section 3.4 on the wave-source duality), following the quasi-static equations

$$-\frac{1}{k_i^2}\Delta\vec{A}_p^i = \vec{j}_p^i \tag{109}$$

$$+\frac{1}{k_i^2}\Delta\vec{A}_c^i = \vec{j}_c^i \tag{110}$$

Due to the infinite number of particles, a huge field is instantaneously generated so that the propagation and confinement equations are driven by the nonlinear terms:

$$-\frac{1}{k_i^2}\Delta\vec{A}_p^i + \left[\sum_i \left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]\vec{A}_p^i = \vec{j}_p^i \tag{111}$$

$$+\frac{1}{k_i^2}\Delta\vec{A}_c^i + \left[\sum_i \left(\vec{A}_p^i + \vec{A}_c^i\right)^2\right]\vec{A}_c^i = \vec{j}_c^i \tag{112}$$

At this step, the nonlinear terms overhead any other force (weak and strong interactions as well as gravitation and electromagnetic forces have not yet emerged) and begin displace the sources terms and especially the particles and anti-particles from their original positions $\vec{r}_p$ and $\vec{r}_c$: the Big-Bang has been ignited.

The following epoch is characterized by huge energies. Creation and annihilation of source terms are supposed to occur everywhere in this ultra-dense medium and extreme collisions between particles take place. As a consequence, particles and anti-particles acquire strong phase shift so that they are no more assigned to a given phase 0 or π as suggested by the $\oplus$ and $\ominus$ signs but may acquire any phase ϕ_0. In that sense, ***there is no more matter and anti-matter in the universe but rather a continuous spectrum of phases ranging from*** 0 ***to*** 2π. Yet, in any new creation of particle/anti-particle pairs the latter do have a π phase shift, but have no more a phase relation with the initial $\oplus$ and $\ominus$ phases of the Big-Bang. This matter/anti-matter "mixing" will have utmost consequences in the structure formation as discussed latter.

As shown in Eqs. 111 and 112 , the ***retardation effects given by the temporal derivatives are negligible compared to the nonlinear terms. Hence, nothing limits the expansion rate of the ultra-dense matter due to the repulsive nonlinear potential of the propagation equation (Eq. 111): highly superluminal speeds [20] are allowed. The inflation mechanism has started*** [1]. Direct interactions related to the near-field (quasi-static) part of the vector potentials have emerged and begins to structure the early universe. In this scenario, the ultra-fast expansion takes place until the density has sufficiently decreased, giving a natural end to the inflation epoch [1]. Since this epoch is the reign of non-linearity its end can be pretty fast as the amplitude of the vector potentials decreases.

Even if the inflation has stopped, the matter still continue to expand. As a consequence, the vector potential amplitude reach a sufficiently low threshold so that ***the nonlinear terms undergo a transition from "parabolic" to "Mexican hat"-like potentials (see fig. 4). Spontaneous symmetry breaking [1, 6] has started***. Almost at the same time, the retardation effects are no more negligible and the full propagation and confinement equations hold:

$$\left[\frac{\partial^2}{\omega_i^2 \partial t^2} - \frac{1}{k_i^2}\Delta\right]\vec{A}_p^i + \left[\sum_i \left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - 1\right]\vec{A}_p^i - \vec{A}_c^i = \vec{j}_p^i \tag{113}$$

$$\left[\frac{\partial^2}{\omega_i^2 \partial t^2} + \frac{1}{k_i^2}\Delta\right]\vec{A}_c^i + \left[\sum_i \left(\vec{A}_p^i + \vec{A}_c^i\right)^2 - 1\right]\vec{A}_c^i - \vec{A}_p^i = \vec{j}_c^i \tag{114}$$

These ***retardation effects are responsible for the far-field behavior of the vector potentials and thus for the appearance of the weak and strong forces (see sections 3.6.2 and 3.6.3). The time of δ-quarks and composite boson has come***. δ-electron, neutrino, protons and neutrons form by combining propagation and confinement potentials (see Table 3) and pairing spins, colored charges and magnetons (see section 4.2). Nuclear reactions producing helium and heavier nuclei quickly follow [1].

The last crucial step in the particle production is the δ-photon/graviton decoupling. It requires very weak interactions between these pure bosons and the composite particles (see section 3.4). If this condition is satisfied, ***δ-photons/gravitons undergo a transition from spherical modes to plane waves as shown in Table 4. This corresponds to the cosmic background birth*** [1]. This pure boson back-ground presents three fundamental properties:

- first, E-modes and B-modes are naturally included in the present theory: E-modes corresponds to the longitudinal pure bosons, i.e. to the δ-gravitons, and the B-modes to the transverse pure bosons, i.e. to the δ-photons. ***These E-modes might be viewed as a proof of existence of the longitudinal δ-gravitons introduced in the present theory. More precisely, they might justify a posteriori the modification of Maxwell's equations performed in section 3.1 in order to derive the driving equations of the DuDy theory***.

- second, the cosmic microwave background finds its very origin in the Big-Bang and is therefore expected to hold a memory of the Big-Bang physical properties. One of them was the required primordial symmetry breaking expressed by Eq. 108. As a consequence, $\tilde{\mathbf{r}}_\mathbf{p} - \tilde{\mathbf{r}}_\mathbf{c}$ ***gives a privileged axis to universe. This is compatible with the alignment of the multipoles observed in the cosmic microwave background and known as the "axis of evil"*** [1, 24]. If true, this would justify the Big-Bang scheme proposed in the present work.

- third, in the present DuDy based on Galilean transformation, ***the cosmic microwave background do have a rest frame corresponding to the frame of the Big-Bang***. This clearly violates the Lorentz invariance which has been accepted as the rule in many theories. It is however in agreement with observations showing that our local group of galaxies appears to be moving at ≈630 km/s relative to the reference frame of the cosmic microwave background radiation [1]. If not fortuitous, this agreement might support the use of Galilean transformation, which was an audacious postulate in the present theory.

Time as come for the formation of macro-structures and, as announced previously, the matter/anti-matter "mixing" will play a major role. If there is not more really matter and antimatter that forms some separate blocks in universe, but rather a smooth, continuous transition from $\phi_0 = 0$ to 2π $[2\pi]$, particles with nearly the same phase undergo gravity while particles having roughly a π phase shift repulse themselves trough anti-gravity. If correct, the void formation proposed in section 5.4 has to be slightly updated. As sketched in Fig. 9, visible matter accumulates on walls and filaments in order to minimize the repulsive anti-gravity, leading to regions whom averaged phases

$\langle\phi_0\rangle$ differ by π $[2\pi]$. This is responsible in the present theory for the apparition of voids where matter is rarefied [23]. On the other hands, gravity exerts an attractive force along filaments or walls inducing a global displacement of galaxies along them and creating large structures. This interpretation is corroborated by the clustering observed in universe [23].

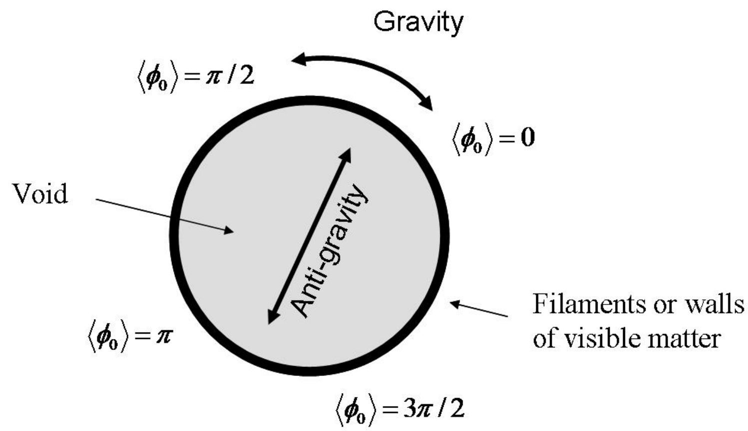

Figure 9: Large scale structures in universe including here filaments, walls and a single void. The average phase of matter varies continuously from $\phi_0 = 0$ $[2\pi]$ *to* 2π $[2\pi]$ *along the filaments or walls.*

The slightly randomized phases around an averaged value at a given position of universe might also explain the matter stability or in other words, why black-holes are the exception and not the rule. In the present theory, black-holes are described by Bose-Einstein condensates (see section 5.1). This means that all particles forming a black-hole must be in the same quantum state. This is clearly not the case if one accepts the concept of particle to particle phase fluctuation. However, in heavy stars where the compression induced by gravity is strong enough, collisions might lead to "phase exchange" between particles since the collision energies will no more be negligible compared to the rest energies. ***Phase exchanges occur over many particles until a pair of particles get exactly the same phase and produce a massive boson as explained in section 5.1. Repeated many times, this mechanism leads to a kind of chain reaction since a high number of bosons further stimulates the creation of identical particles. Black-hole formation has started***, accompanied with a collapse of matter since the fermion-fermion repulsion has disappeared (see section 3.2). Conversely, the phase exchange might also lead to particles having opposite phases and therefore capable of annihilating or repulsing themselves through anti-gravity. ***This dual process, producing a huge number of pure bosons and large repulsive forces is believed to be responsible for the ejection of the remaining matter, leading to a supernova***. The exact process highly depends on the visible/dark matter content as well as their phase distribution, leading to a large variety of final structures, including quasars and blazars (see section 5.2).

There is finally a by product of black-holes formation that has not yet been discussed: the ultra-high-energy cosmic ray [1]. These particles carry a huge energy, exceeding in rare events 10^{20} eV. They might be due, in the present theory, to the ejection of a small Bose-Einstein condensate formed at the periphery of the black-hole during its collapse and the subsequent explosion. As evidenced in Eq. 94, derived for $l = 0$ black-holes, there is a clear connection between the non-linear confinement equation, the nonlinear Schrodinger equation [13] and the Gross-Pitaevskii equation [14]. If these equations are likely to describe Bose-Einstein condensates, they are also known to account for solitons. Hence, ***massive bright solitons can be generated in the present theory, resulting in high-energy rays***. Appropriate simulations are required to test if these solitons can quantitatively explain the ultra-high-energy cosmic rays observed in universe [1].

5.6 Arrow of time

Considering black-hole formation naturally brings to the problem of information loose [15]. In the present theory, black-holes being described by a Bose-Einstein condensates (see section 5.1 and the previous section), all particles have lost their phase information. Even if some matter is ejected during the formation process, it cannot bring its own information plus the information of the particles merged in the condensate. Hence, there is necessary an information loose. However, as discussed in section 5.3, the counter-part is the formation of a dark-matter halo, being also a Bose-Einstein condensate but dramatically larger, since it is expected to extend far away from the main visible matter in galaxies [1]. Hence, it is not clear if the excursion in the phase space has decreased during the overall black-hole/dark-matter creation process or, in other words, if the entropy of the complete system remains constant or has increased as the second law of thermodynamics requires. The same reasoning can be applied to larger structures where gravity and anti-gravity play opposite roles: while gravity tends to form "condensed" macro-structures, anti-gravity push them apart leading to an ever expanding universe with ever denser localized structures. Following this idea, the universe fate is foreseen as a Big-Freeze in the present theory, but does not tell how entropy vary.

Yet, the second law of thermodynamics does hold at our scale. Is there a potential interpretation in the context of the DuDy theory? Following the Boltzmann's H-theorem and its Loschmidt's paradox [25], the derivation of the second law of thermodynamics from differential equations requires a further assumption, namely that the micro-states composing the system are uncorrelated. So the central question is to understand why micro-states described by time reversible equations undergo decoherence, as it turns out to be the origin of irreversibility. This might be explained in the present theory by a very last property of the vector potentials that has been disregarded up to now: the oscillating far-field component associated with the n_l functions (see Table 5). This oscillating contribution scales as $1/r$ at large distances, meaning that it fills the entire space (at least for infinitely long-lived particles as δ-protons and δ-electrons). Hence, the superposition of the vector potentials from all particles leads to a speckle highly fluctuating in time and space due to the intrinsic oscillating frequencies, the arbitrary phases ϕ_0 and the motion of particles. ***This additional back-ground speckle induces a source of decoherence and therefore of irreversibility [25]. It is further assumed to explain phenomena such as spontaneous emission [11] in the context of the present theory***. More precisely, the definition of (nonlinear) creation and annihilation operators adopted in section 2.2 and 3.4 leads to a vanishing field and energy for the vacuum. A side effect is the suppression of the vacuum fluctuations, which are an essential feature in quantum theories. The background speckle reintroduces in some sense the concept of fluctuating fields and therefore the possibility of spontaneous emission, which also gives an arrow of time.

6 Conclusion

In this new theory, many concepts have been revisited or introduced. This includes the definition of 1) nonlinear creation and annihilation operators, leading to a vanishing vacuum energy, 2) fermions and bosons discriminated by the r-dependence of their vector potentials, 3) spins, as being associated with time dependence and 4) colored charges and masses merged in dual entities. But more importantly, new driving equations have been proposed to father propagation and confinement behaviors. This has lead to unique properties such as 1) a revised wave-particle duality, where the creation and annihilation operators apply only on the source terms, 2) an interpretation of the fundamental forces arising either from direct fermion-fermion coupling or indirect coupling mediated by pure bosons. Fundamental properties including confinement, hierarchy between gravitation and electromagnetic interactions, and finally parity violation for the weak interactions have naturally emerged along with candidates for the elementary particles of the Standard Model. The formation of color-less composite particles and generations of particles have been obtained by introducing nonlinear coupling between elementary particles, leading in the latter case to a self trapping mechanism. More surprisingly, the photon wave-function and composite gravitons have also emerged while applying these nonlinear potentials to pure bosons. Owing to these promising results, the DuDy theory has been confronted to the cosmology in order to check its robustness. Using the same coupled and non-linear dual equations, massive Bose-Einstein condensates and related bright solitons have been proposed as candidates for black-holes and ultra-high-energy cosmic rays. Including new jet-particles, quasars- and blazars-like solutions can be generated. Their astonishing comparison with existing structures in universe might be viewed as a proof of existence of technicolor quarks hypothesized previously. Without adding any new physics, non-baryonic particles can be anticipated. In specific circumstances, some of these particles lead to “dark” Bose-Einstein condensates as a by product of a black-hole formation. Due to a repulsive potential, they are associated with a density deep in the central region and are therefore anticipated to generate Dark matter halos. The investigation of dark-energy has necessitated to revisit the equivalence principle in stating that the gravitational mass might be negative while the inertial mass, corresponding to the interaction of a particle with its wave companion, is always positive. With this, the concept of anti-gravity have been actualized and suggested to be responsible for the dark-energy behavior, explaining at the same time the matter/anti-matter imbalance, the formation of voids and the accelerating expansion of universe with the colorful picture of an expansive foam. The universe time-line has finally been analyzed trough the prism of the present theory. Finding potential scenarios to the Big-Bang and the inflation mechanisms has required 1) to introduce a spontaneous generation of source terms leading to a primordial anisotropy of universe and 2) to consider superluminal motion as allowed by the nonlinear terms in the propagation and confinement equations. With this, specific features shared with the cosmic microwave background have been obtained such as the existence of a rest frame, of E- and B-modes corresponding here to gravitons and photons respectively, and of a so-called “axis of evil”. Finally, the far-field properties of the vector potentials have been discussed in connection with the second law of thermodynamics: they produces a fact oscillating speckle foreseen as being a potential source of local irreversibility, while the driving equations are time-reversible.

If the DuDy theory seems capable of accounting for unsolved problems in physics [3], the present work requires an in-depth crosscheck by specialists and adequate simulations since many interpretations have been proposed but awaits for quantitative confrontations with other theories and measured data. It corresponds to a huge work that I cannot do alone. For this reason, I've decided to reveal this theory as is of 2015, even if it might still contain inconsistencies and does require relevant references (see the following section). But the way I went through this theory since 2003 has convinced me that it does have a predictive character, and will help finding new physical properties.

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